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Problem Solving

Sets of Numbers

Ari is comparing the density of some common substances. Density measures how compact a substance is. Use the data in the table for Exercises 1–3.

1. Ari begins by ordering the densities from least to greatest. Write his ordered list.

2. Should Ari use roster, interval, or set-builder notation to represent his data? Why?

3. Which subset of the real numbers best describes the densities given in the table? Choose from \( \mathbb{R} \), \( \mathbb{Q} \), \( \mathbb{Z} \), and \( \mathbb{N} \).

Choose the letter for the best answer.

4. Jean wrote the possible readings on the speedometer of her new car in set-builder notation. Which could be what she wrote?
   A \( \{x \mid 0 > x > 120\} \)
   B \( \{x \mid x = 10n \text{ and } n \leq 12\} \)
   C \( \{10, 20, 30, 40, 50, \ldots, 120\} \)
   D \( \{x \mid 0 \leq x \leq 120\} \)

5. Members of the Booster Club are designing a calendar of the school year to sell as a fund raiser. They begin by making a roster of the possible number of days in a month. Which shows the roster?
   F \( \{30, 31\} \)
   G \( \{x \mid 28 \leq x \leq 31\} \)
   H \( \{28, 29, 30, 31\} \)
   J \( \{x \mid x \geq 28 \text{ and } x \in \mathbb{N}\} \)

Tell whether each statement is true or false. If false, give a counterexample.

6. Trish and Alex are comparing the populations of different cities in Texas. Alex says the population of a city is always an integer.

7. Neil and Sandy are cutting out circles as decorations. Sandy comments that the distance around a circle is always a rational number.
Problem Solving

Properties of Real Numbers

Three friends eat together at a restaurant. Their bill is shown at right. Use mental math for Exercises 1–4.

1. Luke and Willy split one order of the Cajun boil, and they each have a glass of milk. What is Luke's cost for his food and drink?

2. Laska has a bowl of gumbo and a salad. What is her share of the 8.5% sales tax?

3. The group decides to leave double the sales tax as the tip and to divide that amount evenly among themselves. What is Willy's share of the tip?

4. Explain how you would use mental math and the subtotal to find the amount of a 20% tip.

A music store advertises CDs at 15% off the marked price. There is a 6% sales tax added for each purchase. Use mental math to help you choose the letter for the best answer.

5. Todd buys a two-disk set marked $14.95 and a single CD marked $10.95. What is the total of his bill?
   A $23.34
   B $24.90
   C $26.01
   D $27.45

6. Hedy buys 6 CDs marked “3 for $25.” What is her total bill?
   F $41.60
   G $42.50
   H $45.05
   J $50.50

7. Pedro’s total before tax is $66.00. How much tax does he pay?
   A $9.96
   B $6.60
   C $3.96
   D $3.36

8. Gail and Pam decide to share the cost of a three-disk set marked $35.00. What is Pam’s share of the total?
   F $14.50
   G $15.77
   H $29.75
   J $31.54
Problem Solving

Square Roots

A downtown public park has a design of three square fountains. The fountains are arranged so that they create a perspective illusion. The area covered by the largest fountain is 144 square yards. Use this information for Exercises 1–3.

1. The area covered by the smallest fountain is one-fourth the area covered by the largest fountain. What is the side length of the smallest fountain?

2. How does the side length of the smallest fountain compare to the side length of the largest fountain?

3. The area covered by the middle fountain is \(2 \frac{1}{4}\) times the area covered by the smallest fountain. What is the side length of the middle fountain?

There is a rectangular vacant lot between Ken’s house and his school. The dimensions of the lot are 125 ft by 35 ft. Choose the letter for the best answer.

4. Instead of walking along the length and width of the lot, Ken sometimes takes a shortcut and walks along the diagonal. What is the difference in distance between Ken walking along the sidewalk or taking a shortcut?
   - A 30 ft
   - B 130 ft
   - C 160 ft
   - D 190 ft

5. Ken gets a job mowing the lot. He charges $30 to mow a square lot 35 feet long. At the same rate, about what will he charge to mow this lot?
   - F $60
   - G $84
   - H $107
   - J $1050

6. The owner of the lot divides it into 3 garden plots. Two of the plots have an area of 1225 square feet each. What could be the dimensions of the third plot?
   - A 125 ft by 125 ft
   - B 75 ft by 35 ft
   - C 55 ft by 35 ft
   - D 35 ft by 35 ft

7. Ken has a job mowing a park that is made up of 6 congruent square areas separated by paths. The total area of the park is 1734 square yards. What is the approximate length of the side of each square?
   - F 145 yd
   - G 132 yd
   - H 42 yd
   - J 17 yd
Problem Solving
Simplifying Algebraic Expressions

To find out how much water a dripping showerhead wastes, Marisa catches the drips in a measuring cup. She collects a cup of water in 6 minutes.

1. Write and simplify an expression for the number of cups of water wasted in $t$ minutes.

2. How much water does this showerhead waste in an hour?

3. How many minutes will it take for this showerhead to waste a gallon of water?

4. Write expressions for the amount of water, in cups and in gallons, that the dripping showerhead wastes in $d$ days.

Choose the letter for the best answer.

5. Van budgets $12 a day for groceries for weekdays and $15 a day for weekend days. Which expression could be used to find his grocery budget for $w$ weeks?
   - A $w(12 \cdot 5 + 15 \cdot 2)$
   - B $7w(12 + 15)$
   - C $12w + 15w$
   - D $17w$

6. The Spanish Club is planning to make a quilt for the annual fund-raiser. The quilt design includes 30 blue triangles, each with a base of 4 inches and a height of $h$ inches, and 20 blue squares that are each $s$ inches on a side. Which expression could be used to find the total area of blue fabric needed?
   - F $(30 \cdot 4)h + 20s$
   - H $60h + 20s^2$
   - G $120h + 20s$
   - J $240sh$

7. Susanne wrote the expression $25 + 0.15m$ for the monthly cost of her cell phone where $m$ is the number of minutes she uses. What will her bill be this month if she makes 55 minutes of calls?
   - A $8.25$
   - B $33.25$
   - C $63.25$
   - D $80.00$

8. The width of a rectangle is $3g^2$. The length of the rectangle is $h^2 - 2h + 5$. Which represents the area of the rectangle?
   - F $3g^2 + h^2 - 2h + 15$
   - G $3g^2h^2 + 6gh - 15g^2$
   - H $18g^2h^2 - 6gh$
   - J $3g^2h^2 - 6g^2h + 15g^2$
Problem Solving

Properties of Exponents

Joan made a presentation to her technology class about the history of the Internet. Use the data in her table for Exercises 1–6.

1. About how many million Internet users were there in the world in 1997?

2. When did the estimated number of Internet users in the United States show the greatest increase?

3. What was the first year the number of Internet users in the United States exceeded 10 million?

4. During which year was there about the same number of Internet users in the United States as in the world?

5. During which two years was the number of Internet users in the world almost double the number of Internet users in the United States?

6. By what factor did the number of Internet users increase in the United States from 1992 to 2002?

Choose the letter for the best answer.

7. In 1790 the population of the United States was about $3.9 \times 10^6$. By 2000 the population had grown to around $2.8 \times 10^8$. By what factor did the population increase?
   A 720
   B 72
   C 7.2
   D 0.72

8. Lee is packing three congruent storage boxes. Each box is $2b^3$ high, $b^4$ long, and $3b^{-2}$ wide. Which expression gives the total volume of the 3 boxes?
   F $6b^5$
   G $4b^7$
   H $18b^5$
   J $12b^7$
In order to make a nutrition plan, Richard wants to compare different types of milk. Use the table for Exercises 1–7.

1. Is the relation from calories to saturated fat a function? Explain why or why not.

2. Is the relation from calories to carbohydrates a function? Explain why or why not.

3. Is the relation from carbohydrates to calories a function? Explain why or why not.

Choose the letter for the best answer.

4. Richard is drawing graphs of some of the relations from the table above. Which of these graphs fails the vertical-line test if he graphs the data as follows?
   - A column B along the x-axis, column C along the y-axis
   - B column D along the x-axis, column B along the y-axis
   - C column D along the x-axis, column C along the y-axis
   - D column C along the x-axis, column B along the y-axis

5. For the function (B, D) that relates calories to saturated fat, which column shows the domain?
   - F column A
   - G column B
   - H column C
   - J column D

6. Which column shows the range of a function that relates the type of milk to the number of calories?
   - A column A
   - B column B
   - C column C
   - D column D

7. Richard makes a mapping diagram from each type of milk to the number of students in his class of 25 who prefer that type of milk. Which is the best statement about this diagram?
   - F It is a relation, but not a function.
   - G It is a function, but not a relation.
   - H It is a function and a relation.
   - J It is not a relation or a function.
Problem Solving

LESSON 17  Function Notation

Juan is analyzing cell phone plans. The graph shows two plans he is considering. Use the graph for Exercises 1–4.

1. For which value of \(x\) does each function have a value of $40?

2. The graphs of the functions cross at \(x = 150\). Explain what this represents.

3. Use function notation and estimation to represent the value of each function for 200 minutes.

4. Juan expects to use about 300 minutes per month. Which plan should he buy? Why?

In September, Harley puts $1035 that he earned during the summer in a bank account to use during the school year for his personal expenses. He budgets \(d\) dollars a month for expenses. Choose the letter for the best answer.

5. Which shows a function representing the amount left in his account after 4 months?
   A  \(f(d) = $1035 - 4d\)
   B  \(f(d) = $1035 - d\)
   C  \(f(d) = ($1035 - 4)\)
   D  \(f(d) = \frac{1035}{4d}\)

6. Harley writes the function \(g(a) = \frac{1035}{9} - a\) to show his monthly budgeted amount remaining in a month when \(a\), the actual amount he spends, is less than the amount of his budget. What is the value of this function for a month when he spends $87.50?
   F  $12.50
   H  $115.00
   G  $27.50
   J  $202.50

7. Fay uses the function \(f(x) = \frac{3}{2}x + 1\) to find the number of boxes of tile to buy for each 10 square feet of floor. How many boxes of tiles does she need to cover 600 square feet?
   A  901 boxes
   B  401 boxes
   C  91 boxes
   D  41 boxes

8. Rasheed uses the function \(f(e) = 4e\) to find the distance around a square barbecue pit. What is the length of the side of the pit that has a perimeter of 55.2 ft?
   F  13.8 ft
   G  27.6 ft
   H  110.4 ft
   J  220.8 ft
Harry is working on a budget for a concert. The graph shows the total cost of renting the hall. A cleaning fee of $40 for each rental is included in the graph. Use the graph for Exercises 1–6.

1. What is the cost of renting the hall for 2 hours? for 3 hours? for 6 hours? for 7 hours?

2. What is the rate per hour not including the cleaning fee if Harry rents the hall for up to 3 hours?

3. What is the rate per hour after the first 3 hours?

4. Describe the effect on the graph if the cleaning fee were changed to $25.

5. The managers decide that the minimum time for which the hall can be rented is 3 hours. Describe the effect this change would have on the graph above. How would the range change?

6. The Art Center gives Harry a graph showing its charges. This graph is the same shape as the graph above, but every point has been translated up 10 units. What would be the effect on Harry's budget if he chose to have the concert at the Art Center?

Choose the letter for the best answer.

7. Martha's profits from her bagel store last year were $0.35 per dozen bagels sold. This year her profits decreased 10%. What kind of transformation does this represent?
   - A  vertical compression
   - B  vertical stretch
   - C  horizontal compression
   - D  horizontal stretch

8. Shana drew the graph for a quadratic function. Then she did a horizontal stretch of the curve. Which transformation did she perform?
   - F  \((x, y) \rightarrow (x, ay); \ |a| > 1\)
   - G  \((x, y) \rightarrow (bx, y); \ 0 < |b| < 1\)
   - H  \((x, y) \rightarrow (x, ay); \ 0 < |a| < 1\)
   - J  \((x, y) \rightarrow (bx, y); \ |b| > 1\)
Katy and Peter are writing a paper about the history and use of cell phones. They make a graph of the data in the table. They want to determine the parent function for the graph.

<table>
<thead>
<tr>
<th>Cell Phone Subscribers in the United States (estimated in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
</tr>
<tr>
<td>1992</td>
</tr>
<tr>
<td>1993</td>
</tr>
<tr>
<td>1995</td>
</tr>
<tr>
<td>1996</td>
</tr>
</tbody>
</table>

1. Peter wants to compare the graph to the function \( f(x) = 7x + 2 \).
   How would the graph of \( f(x) = 7x + 2 \) compare to its parent function \( f(x) = x \)?

2. What is the value \( f(x) = 7x + 2 \) for 1996, when \( x = 6 \)? Does that point fit the graph? Try some other values of \( x \) for the function \( f(x) = 7x + 2 \). How well do the results fit the range of the graph?

3. Katy wants to compare the graph to the function \( f(x) = x^2 + 5 \).
   How would the graph of \( f(x) = x^2 + 5 \) compare to its parent function \( f(x) = x^2 \)?

4. Find the value of \( f(x) = x^2 + 5 \) for 1996, when \( x = 6 \)? Does that point fit the graph? Try some other values of \( x \) for the function \( f(x) = x^2 + 5 \). How well do the results fit the range of the graph?

5. Which parent function and transformation best models these data?
LESSON 2-1
Problem Solving

Solving Linear Equations and Inequalities

Trish keeps track of a leak in her outside water faucet by measuring the depth of the water that collects in a barrel under the faucet. Her results (rounded to the nearest 0.5 centimeter) are shown in the table. Use the data in the table for Exercises 1–3.

<table>
<thead>
<tr>
<th>Water Leak</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Water in Barrel (cm)</td>
<td>3.5</td>
<td>8</td>
<td>12.5</td>
<td>17</td>
<td>21.5</td>
<td>26</td>
</tr>
</tbody>
</table>

1. After the first day, what is the depth of the water that leaks each day?

2. Write an equation for the total depth of the water in the barrel, \( y \), in terms of the number of days that the faucet leaks, \( x \).

3. If this pattern continues, and assuming no evaporation of the water in the barrel, when will the depth of the water be greater than 45 centimeters?

Choose the letter for the best answer.

4. A ream of computer paper is 2.1 in. high. Which inequality can be used to find the maximum number of reams that will fit in one stack between two shelves that are 1.5 ft apart?
   - A \( 2.1x \leq 1.5 \)
   - B \( \frac{2.1}{x} \leq 1.5 \)
   - C \( 2.1x \leq 18 \)
   - D \( \frac{2.1}{x} \leq 18 \)

5. Kevin is 5 years older than Keith, but 3 years younger than Kara. The total of their ages is 49. Which equation can be used to find Keith's age?
   - A \( x + (x + 5) + (x + 3) = 49 \)
   - B \( x + (x - 3) + (x + 8) = 49 \)
   - C \( x + (x + 5) + (x - 3) = 49 \)
   - D \( x + (x + 5) + (x + 8) = 49 \)

Tell whether each problem is solved correctly. If not, explain why and find the correct solution.

6. Ana wants to find a solution to a problem by solving this inequality: \(-3x < 42\). She multiplies both sides by \(-\frac{1}{3}\) and found the solution \(x < -14\).

7. Tyler writes this equation to solve a problem: \(3(b - 4) = 9 + 6b\). He divides both sides of the equation by 3 to get \(b - 4 = 3 + 2b\) so he concludes that \(b = -7\).
Problem Solving

Leon likes to build model sets of his favorite movies. Solve.

1. In *The Incredible Shrinking Man* (1957), the hero shrinks from about 5 feet 10 inches to about 1 inch tall. If Leon starts with a model character that is $8\frac{3}{4}$ inches tall and shrinks by the same proportion, what is the character’s final height?

2. In a scene in *King Kong* (1933), the giant gorilla is about 22 feet tall. An ordinary gorilla is about 5 feet tall. In his model, Leon made the giant gorilla 9 inches tall. How tall would an ordinary gorilla be in his model set?

3. In *The Amazing Colossal Man* (1957), a man of about 6 feet becomes 100 feet tall. How tall will Leon’s character become if he starts with a model 1\(\frac{1}{2}\) inch tall?

4. Leon wants to build a model in a shoe box of the movie *Mothra* (1962), in which a caterpillar with a radius of about \(\frac{1}{2}\) inch grew to have a radius of about 30 feet. He starts with a model of an ordinary caterpillar with a radius of \(\frac{1}{8}\) inch. Will his final model fit in the shoe box? Why or why not?

Choose the letter for the best answer.

5. There are 372 students, or 30\% of the student enrollment, in the 10th grade at Highland School. What is the total student enrollment at the school?

   A 1240
   B 1116
   C 744
   D 37.2

6. In this year’s city election, 23.6\% of the 13,457 registered voters went to the polls. About how many people voted this year?

   A 236
   B 3176
   C 9877
   D 317,585

7. Elijah is 5 feet 6 inches tall. He casts a shadow that is 20 feet long. He is standing next to a palm tree that casts a shadow that is 76 feet 6 inches long. How tall is the pine tree to the nearest foot?

   A 19 ft
   B 21 ft
   C 25 ft
   D 278 ft

8. How many inches are there in 100 miles?

   A \(6.336 \times 10^3\) in.
   B \(6.336 \times 10^4\) in.
   C \(6.336 \times 10^5\) in.
   D \(6.336 \times 10^6\) in.
LESSON 2-3

**Problem Solving**

**Graphing Linear Functions**

Solve

1. Nathan made a table to record the balance in his savings account when he made a deposit every other month.

<table>
<thead>
<tr>
<th>Month</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance ($)</td>
<td>575</td>
<td>810</td>
<td>1025</td>
<td>1280</td>
<td>1545</td>
<td>1850</td>
</tr>
</tbody>
</table>

Is this data set linear? How do you know?

2. Sally runs a landscape service business. The table shows her fee schedule.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($)</td>
<td>8</td>
<td>14</td>
<td>20</td>
<td>26</td>
<td>32</td>
<td>38</td>
</tr>
</tbody>
</table>

a. Why is the data set linear?

b. Find the slope of the line that passes through the points.

c. Graph these data.

d. Estimate the cost for 9 hours of landscape services.

Choose the letter for the best answer.

3. Jan built a skateboard ramp from her back porch to the ground. The porch is 30 inches above the ground. The ramp extends 9 feet from the base of the porch. Find the slope of the ramp.

A 3.6 \hspace{1cm} C 0.3
B 3.33 \hspace{1cm} D 0.278

4. When Rafiq left home on a business trip he noted that the odometer on his car read 47,823. He drove 3 h 15 min and then noted that the odometer read 48,017. Find his average speed in miles per hour.

A 55.6 \hspace{1cm} C 61.6
B 59.7 \hspace{1cm} D 63.5
LESSON Problem Solving

2-4 Writing Linear Functions

Solve.

1. As an image consultant, Antonio is well paid by his celebrity clients. For 20 hours of work, his fee is $1625. He charges $3500 for 45 hours of his time.

a. Express his fee as a function of his hourly rate.

b. What does the $y$-intercept represent?

c. What is the fee for 40 hours of his time?

2. A child’s cough medicine has a dosage table on the package.

<table>
<thead>
<tr>
<th>Medicine Dosage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child’s Weight (kg)</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>28</td>
</tr>
<tr>
<td>30</td>
</tr>
</tbody>
</table>

a. Express the dosage in milliliters as a function of the child’s weight in kilograms.

b. Graph the function.

c. Find the dosage for a child who weighs 22 kg.

Choose the letter for the best answer.

3. Kayla’s cell phone plan charges a fee of $25 per month and $0.25 per minute. She just received a notice that the fee is being increased to $35 per month. Which equation models her new cell phone plan?

A $y = 0.25x + 25$
B $y = 0.35x + 35$
C $y = 0.25x + 35$
D $y = 0.35x + 25$

4. Henry drew 2 lines on a coordinate grid. The red line passes through points $(-2, -4)$ and $(2, -2)$. The blue line is perpendicular to the red line and includes point $(-2, 8)$. What is the equation of the blue line?

A $y = \frac{1}{2}x - 3$
B $y = -2x + 4$
C $y = -\frac{1}{2}x - 3$
D $y = 2x + 4$
Problem Solving

2-5 Linear Inequalities in Two Variables

Mr. and Mrs. Zaragosa are planning a landscape garden for their new house. They have set a budget of $200 for native grasses, at $12 each, and flowering plants, at $8.50 each.

1. Let \( n \) be the number of native grasses and \( p \) be the number of flowering plants. Write an inequality for the number of each that they can buy.

\[
12n + 8.50p \leq 200
\]

2. Find the intercepts of the boundary line.

- \( n \)-intercept: \( 16.6 \) (rounded to one decimal place)
- \( p \)-intercept: \( 23.5 \) (rounded to one decimal place)

3. Graph the inequality on the coordinate plane.

4. Define the domain for variables \( n \) and \( p \).

Positive whole numbers

5. Should the boundary line be dashed or solid? Why?

Solid, because the total cost could be equal to $200

6. What is the solution region on the graph? How do you know?

The solution region is the area below the line, because they cannot spend any more than $200.

7. Use your graph to determine if they can buy 10 flowering plants and 15 native grasses. How do you know?

No; Possible answer: because the point (10, 15) is not in the shaded region of the graph, so it is not a solution.

8. What is the greatest number of grasses they can buy if they want to buy at least 5 flowering plants?

13

9. Use your graph to estimate the number of each they could buy if they want the same number of each type of plant.

9 of each type.

Choose the letter for the best answer.

10. What is the greatest number of grasses they can buy if they have already bought 8 flowering plants?

- A 8
- B 9
- C 10
- D 11

11. What is the greatest number of flowering plants they can buy if they decide to buy a dozen native grasses?

- A 6
- B 7
- C 8
- D 9
Problem Solving

Transforming Linear Functions

The students in Ms. Hari’s English class are planning to print a booklet of their creative writings. Use the table of publishing prices.

1. The students decide to print a booklet containing black and white text only. Write a function, \( C(p) \), to show the cost of printing a booklet of \( p \) pages with a cover that also has text only.

2. Julie wants the booklet cover to have a color graphic. Write a new function, \( J(p) \), to show this cost for a booklet of \( p \) pages.

3. What is the slope of each function? What does the slope tell you about the relationship of the lines?

4. What is the \( y \)-intercept of each function? What is represented by the \( y \)-intercept?

5. Describe the transformation that has been applied to the graph by the decision to change the cover.

6. Oscar suggests that the booklet have 30 pages, one for each person in the class. What is the cost of printing 50 booklets, using the function \( J(p) \)?

7. Lee writes a function for the cost of \( p \) pages, all in color, with a plain text cover. What transformation does this apply to the graph of \( C(p) \)?
   A. Horizontal stretch
   B. Horizontal compression
   C. Vertical stretch
   D. Vertical compression

8. Tina finds a printer who will print text pages at $0.25 a page, with a color cover for $2.00. Using this printer, what is the cost of 50 booklets of 30 pages each?
   A. $950
   B. $725
   C. $600
   D. $475

Choose the letter for the best answer.
LESSON 2-7 Curve Fitting with Linear Models

As a science project, Shelley is studying the relationship of car mileage (miles per gallon) and speed (miles per hour). The table shows the data Shelley gathered using her family’s hybrid vehicle.

<table>
<thead>
<tr>
<th>Speed (miles per hour)</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mileage (miles per gallon)</td>
<td>34.0</td>
<td>33.5</td>
<td>31.5</td>
<td>29.0</td>
<td>27.5</td>
</tr>
</tbody>
</table>

1. Make a scatter plot of the data. Identify the correlation.

2. Sketch a line of best fit on the graph.

3. Use two points on the line to find the slope.

4. Use the point-slope form to write an equation that models the data.

5. Use a graphing calculator to plot the data. Find the value of the correlation coefficient \( r \).

6. What does the value of \( r \) tell you about the data?

7. What equation do you find with the calculator for the line of best fit?

Use the equation you wrote in Exercise 3. Choose the letter for the best answer.

8. Predict the mileage for a speed of 55 miles per hour.
   - A 30
   - B 34
   - C 39
   - D 46

9. Predict the speed if the mileage is 28 miles per gallon.
   - A 32
   - B 35
   - C 67
   - D 75
Problem Solving

Solving Absolute-Value Equations and Inequalities

Gita’s science class is making a set of posters about North American wildlife. The table shows some of the data collected.

1. What is the center of each weight group?
   a. \(W_1\)
   b. \(W_2\)
   c. \(W_3\)

2. Express each weight group as an absolute-value expression.
   a. \(W_1\)
   b. \(W_2\)
   c. \(W_3\)

3. Write inequalities to show the amount of food required each day for animals in each weight group.
   a. \(W_1\)
   b. \(W_2\)
   c. \(W_3\)

4. Gita wants to use the term *disjunction* or *conjunction* on her poster showing the inequalities. Which term should she use? Why?

5. Les includes the following on his poster:
   Solve this equation to find the number of kilograms of food consumed each day by an animal in one of the weight groups:
   \[ |f - 7.2| = 3.3. \]
   Find the solution.

6. Write an absolute-value inequality to represent the maximum weight difference between a grizzly bear, \(g\), and a black bear, \(b\).
Problem Solving

LESSON 2-9

Absolute-Value Functions

Linette and Dylan are observing people passing by an outdoor art exhibit in the park. The graph shows the average path of a person walking past the exhibit at a rate of 1 block per minute.

1. Write an absolute-value equation that represents the graph. Express the distance in blocks from the exhibit, \( D(t) \), as a function of minutes \( t \) before and after arriving at the exhibit.

2. Find the value of \( D(t) \) for \( t = 3 \) and for \( t = -3 \). What do these values mean about the position of a person walking past the exhibit?

3. What is the domain of the function?

4. What is the range of the function?

5. A person rides past the exhibit on a bicycle at a rate of 5 blocks per minute.
   a. Write an absolute-value function, \( C(t) \), to represent the distance the bicycle is from the exhibit at any time, \( t \).

   b. Sketch \( C(t) \) on the graph. Describe the transformation.

   c. Compare the vertex of \( C(t) \) to the vertex of \( D(t) \).
Problem Solving

Using Graphs and Tables to Solve Linear Systems

Solve.

1. After the lesson, Carl takes the wakeboarding class to the Glass Cafe. He pays $26 for 8 large and 4 small juice drinks. A large glass costs $1 more than a small glass.
   a. Write a linear system of equations to find the cost of each size drink.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

   b. Write one equation at the top of each table and complete the table.
   c. What is the cost of each size drink?

2. Sandy rented a jet ski for $95 plus $15 per hour. Pauline rented a jet ski for $80 plus $20 per hour.
   a. Write a linear system of equations to find the number of hours for which the rental cost is the same.

   b. Graph the system.
   c. For what number of hours would Sandy and Pauline pay the same to rent a jet ski?

   d. How much would it cost to rent the jet ski for this amount of time?

Choose the letter for the best answer.

3. Juan started with 50 gallons of water in his pool, and he is filling it at a rate of 10 gallons per minute. His next-door neighbor Sam started with 20 gallons of water in his pool, and he is filling it at a rate of 15 gallons per minute. Which system of equations could you use to find when the pools will contain the same amount of water?
   A \( y = 50 + 15x \)
   \( y = 20 + 10x \)
   B \( y = 50 + 10x \)
   \( y = 20 + 15x \)
   C \( y = 50 - 15x \)
   \( y = 20 - 10x \)
   D \( y = 50 - 10x \)
   \( y = 20 - 15x \)
Problem Solving

LESSON 3-2 Using Algebraic Methods to Solve Linear Systems

Shanae mixes feed for various animals at the zoo so that the feed has the right amount of protein. Feed X is 18% protein. Feed Y is 10% protein. Use this data for Exercises 1–4.

1. How much of each feed should Shanae mix to get 50 lb of feed that is 15% protein?
   a. Write a linear system of equations.
   b. Solve the system. How much of each feed should she mix?

2. Shanae has 15 lb of Feed Y left. She wants to make a mixture that is 12% protein. She needs to know how much of Feed X to use, and how much of the mixture she can make.
   a. Write a linear system of equations.
   b. How much of Feed X should she use?
   c. How much of the mixture will she make?

Choose the letter for the best answer.

3. Raul mixes 12 lb of Feed X with 20 lb of Feed Y. Which equation gives the percent of protein (c) in the mixture?
   A 12(0.18) + 20(0.10) = 32c
   B 32[12(0.18) + 20(0.10)] = c
   C 12(0.18) + 20(0.10) = c
   D [12(0.18) + 20(0.10)]c = 32

4. Alonzo needs to know how much of Feed X and Feed Y to mix to get 25 lb of a mixture that is 12% protein. Which equation can be used as part of a system of equations to find the solution?
   A (0.10 + 0.18)(x + y) = (0.12)25
   B (0.18)x + (0.10)y = (0.12)25
   C 25(0.18 + 0.10) = (0.12)x
   D 10 + 18 = (0.12)25

5. Billie reorders Feed X and Feed Y. Feed X costs $58 per 100 lb. Feed Y costs $45 per 100 lb. The order comes to $470 for 900 lb. How much of each did she order?
   A Feed X: 350 lb; Feed Y: 550 lb
   B Feed X: 400 lb; Feed Y: 500 lb
   C Feed X: 450 lb; Feed Y: 540 lb
   D Feed X: 500 lb; Feed Y: 400 lb

6. Shanae earns $8.00 per hour during the daytime and $9.50 per hour in the evenings after 6 P.M. Last week she earned $314.00 for 37 hours. How many daytime and evening hours did she work?
   A 35 daytime; 2 evening
   B 30 daytime; 7 evening
   C 25 daytime; 12 evening
   D 20 daytime; 17 evening
Problem Solving

3-3 Solving Systems of Linear Inequalities

Marshall and Zack plan a hike-and-canoe vacation in a national park. They plan to hike for $m$ hours at a steady 3 miles per hour and canoe for $n$ hours at 6 miles per hour. They want to travel no more than 8 hours and cover at least 40 miles in a day.

1. Marshall makes a table to find the number of hours they can hike and canoe and still meet their goal.
   a. Complete the table.
   b. What different options do they have in whole numbers of hours of hiking and canoeing while still meeting their goal?

<table>
<thead>
<tr>
<th>Hiking Time (m)</th>
<th>Canoeing Time (n)</th>
<th>Total Miles per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

2. a. Write a system of inequalities to model the conditions.
   \[
   \begin{align*}
   3m + 6n &\leq 100 \\
   4m + n &\geq 40 \\
   m + n &\leq 8
   \end{align*}
   \]
   b. Graph the boundary lines. Shade the areas to show the inequalities and the overlapping region.
   c. Describe how the overlapping shaded region relates to the solution to the inequalities.
   d. Name a point within the solution region.

Choose the letter for the best answer.

3. Which point is NOT in the region that satisfies the goals?
   A (4, 4)  
   B (2, 6)  
   C (1, 6.75)  
   D (0, 7)

4. How could you interpret the point of intersection of the boundary lines?
   A They will travel exactly the same number of hours hiking as canoeing.
   B This represents the only possible solution.
   C It is the only impossible combination of hiking hours and canoeing hours.
   D They will travel exactly 40 miles in exactly 8 hours.
Problem Solving

Linear Programming

At the local fair, Tamara wants to demonstrate her “build-it-yourself” products. She decides to hire some skilled technicians and some students to build her garden chairs. Technicians can build 2 chairs in 8 hours; students can build only 1 chair in 8 hours. She wants at least 1 technician to work with every 3 students, but can find only 6 technicians. She will pay $12 per hour for technicians and $7 per hour for students, and can spend up to $800. What combination of technicians and students can build the greatest number of chairs in an 8-hour day?

1. Write the constraints needed to graph the feasible region.

2. Graph the feasible region.

3. List the vertices of the feasible region.

4. Write the objective function, C, to show the total number of chairs that can be built.

5. At which vertex is the objective function maximized?

6. How many technicians and students should Tamara hire to achieve her goal?

Choose the letter for the best answer.

7. Tamara decides that at least 1 technician should work with every 2 students. How does this change her hiring plan?
   A She should hire only the 6 technicians.
   B She should hire 4 technicians and 7 students.
   C She should hire 6 technicians and 12 students.
   D Her hiring plan will not change.

8. Tamara finds 1 more technician. How does this change her hiring plan?
   A She should hire only the 7 technicians.
   B She should hire 3 technicians and 9 students.
   C She should hire 7 technicians and 2 students.
   D Her hiring plan will not change.
Problem Solving

3-5 Linear Equations in Three Dimensions

To play Space Force, Lily, Alicia, and Van must define a space plane relative to the mother ship at the origin. They choose 5 in the $x$-dimension, 4 in the $y$-dimension, and 10 in the $z$-dimension, and a total space force of 60. The Space Force software creates the plane that they have defined, and places Alicia at the $x$-intercept, Lily at the $y$-intercept, and Van at the $z$-intercept. It then constrains them to move only in the space plane. The objective is to move to the same location using the least number of moves based on each other’s moves.

1. a. Write a linear equation in three dimensions that defines the plane.
   
   _______________________

   b. Complete the table to show the $x$-, $y$-, and $z$-intercept starting positions for each player.

2. Plot the starting locations for each player on the grid and draw the space plane.

3. To play the game, each player moves by defining a change in two dimensions. The Space Force software then uses the linear equation to find the third dimension and relocates the player in the space plane. Find the new location for each player.
   
   a. Alicia changes her location by $(−2, 1, z)$.
      
      _______________________

   b. Lily changes her location by $(x, −10, 2)$.
      
      _______________________

   c. Van changes his location by $(1, y, −2)$.
      
      _______________________

Choose the letter for the best answer.

4. Which point is NOT in the space plane?
   
   A $(0, 6, 3.6)$  
   B $(1, 1.25, 5)$  
   C $(4, 3.4, 2)$  
   D $(8, 2.5, 1)$

5. Which player is the greatest distance from the mother ship at the start of the game?
   
   A Alicia  
   B Lily  
   C Van  
   D They are all the same distance from the mother ship.
Problem Solving

Solving Linear Systems in Three Variables

For an annual violin competition, judges score the musicians in three categories: technique, creativity, and presentation. Each category is worth a percent of the final score. Use the table for Exercises 1–3.

<table>
<thead>
<tr>
<th>Musician</th>
<th>Technique</th>
<th>Creativity</th>
<th>Presentation</th>
<th>Final Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jonathan</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>7.6</td>
</tr>
<tr>
<td>Miguel</td>
<td>9</td>
<td>4</td>
<td>8</td>
<td>7.4</td>
</tr>
<tr>
<td>Travis</td>
<td>6</td>
<td>10</td>
<td>6</td>
<td>7.0</td>
</tr>
</tbody>
</table>

1. Find the value of each category in the judging.
   a. Write a system of equations to represent the data in the table.

   \[ \begin{align*}
   7a + 8s + 8c &= 52 \\
   9a + 4s + 8c &= 56 \\
   6a + 10s + 6c &= 44 \\
   \end{align*} \]

   b. What percent of the final score is based on technique? 
   c. What percent of the final score is based on creativity? 
   d. What percent of the final score is based on presentation?

2. How many more points would Miguel have needed to score for creativity to win the competition?

3. Leesa scored 8 points for technique and 9 points for creativity. Her final score was 7.2. How many points did she score for presentation?

A student-written performance is playing at a local high school.
Rachelle paid $52 for two adult, two student, and one child tickets; RJ paid $56 for one adult, two student, and three child tickets; and Hong-An paid $44 for one adult and four child tickets. Choose the letter for the best answer.

4. Austin wants to know the cost of each type of ticket, \( a \), \( s \), and \( c \). RJ says she can write a system of equations using the data. Which equation is not part of this system?
   \[ \begin{align*}
   &\text{A} \quad 2a + 2s + c = 52 \\
   &\text{B} \quad a + 2s + 3c = 56 \\
   &\text{C} \quad a + 4c = 44 \\
   &\text{D} \quad 2a + s + 2c = 54 \\
   \end{align*} \]

5. Austin solves the correct system of equations. What is the price for each type of ticket?
   \[ \begin{align*}
   &\text{A} \quad \text{Adult: $13; student: $9; child: $7} \\
   &\text{B} \quad \text{Adult: $12; student: $10; child: $8} \\
   &\text{C} \quad \text{Adult: $11; student: $10; child: $10} \\
   &\text{D} \quad \text{Adult: $11; student: $9; child: $8} \\
   \end{align*} \]
Problem Solving

Matrices and Data

According to the United States Census Bureau, Americans are more educated than ever: about 84% of adults over 25 have at least completed high school and about 26% have at least a bachelor’s degree. Olivia is researching whether completing higher levels of education has a payoff in higher lifetime earnings. Use the information in the table to answer the questions.

1. Display the data in the form of a matrix. Write matrix \( P \).

2. What are the dimensions of matrix \( P \)?

3. Olivia wants to show the earnings per month, rather than annual earnings. Write an equation with a scalar product that she can use to change matrix \( P \) into matrix \( M \).

4. Write matrix \( M \). Round each entry to the nearest tenth.

Choose the letter for the best answer.

5. Olivia’s uncle is represented in matrix \( P \) by the entry \( p_{21} \). What are the average annual earnings for this entry?
   A $38.20
   B $45.40
   C $38,200
   D $45,400

6. Olivia’s cousin graduated from high school and works in the summer only. What is the address for the entry that represents her?
   A \( p_{13} \)
   B \( p_{23} \)
   C \( p_{31} \)
   D \( p_{32} \)
Members of the Cooking Club entered the Culinary Challenge. In this contest, the score for each entry is multiplied by an assigned degree of difficulty.

<table>
<thead>
<tr>
<th>Cooking Club Members Scores</th>
<th>Culinary Challenge Degrees of Difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Appetizer</td>
</tr>
<tr>
<td>Beth</td>
<td>25</td>
</tr>
<tr>
<td>Jon</td>
<td>35</td>
</tr>
<tr>
<td>Lupe</td>
<td>20</td>
</tr>
<tr>
<td>Amy</td>
<td>40</td>
</tr>
</tbody>
</table>

1. Display each table as a matrix. Matrix $S$ should show the scores and matrix $D$ should show the degrees of difficulty.

2. Write an equation using $S$, $D$, and product matrix $P$ you could use to evaluate the final scores.

3. Explain how you know that matrix $S$ can be multiplied by matrix $D$.

4. Write the product matrix $P$.

5. Roger is writing a story for the school newspaper about the Culinary Challenge. Explain how he can use $P$ to find the final scores for his story.

6. List the contestants and their final scores, in descending order.
LESSON 4-3

Problem Solving

Using Matrices to Transform Geometric Figures

Sherrill is trying to re-create the pattern of a vintage quilt she saw at an antique store. The shaded parts of the figure show the pattern of the quilt.

1. What directions would you give Sherrill to help her draw triangle $A$ on a grid?

2. a. What transformation can Sherrill use on triangle $A$ to create triangle $B$?
   
   b. What transformation matrix should she use to create triangle $B$?

3. a. What transformation can Sherrill use on triangle $A$ to create triangle $C$?
   
   b. What transformation matrix should she use to create triangle $C$?

4. a. What transformation can Sherrill use on triangle $A$ to create triangle $D$?
   
   b. What transformation matrix should she use to create triangle $D$?

Choose the letter for the best answer.

5. Jesse drew a rectangle represented by

   $R = \begin{bmatrix} 2 & 5 & 5 & 2 \\ -3 & -3 & -5 & -5 \end{bmatrix}$. He added the transformation matrix

   $\begin{bmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \end{bmatrix}$ to $R$ and drew a second rectangle. Then he added the transformation matrix

   $\begin{bmatrix} -3 & -3 & -3 & -3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ to $R$ and drew a third rectangle. Which describes the resulting figure?

   A. Rectangle
   B. Irregular hexagon
   C. Square
   D. Irregular octagon

6. Tina drew rectangle $F$ with vertices at $(0, 0)$, $(0, 5)$, $(3, 5)$, and $(3, 0)$. She wants to transform $F$ into a rectangle that is 6 units wide and 10 units long with the center of the rectangle located at the origin. Which list of transformations will accomplish that?

   A. Rotate $F$ 90° clockwise, rotate $F$ 90° counterclockwise, translate $F$ 5 units left and 3 units down
   B. Reflect $F$ over the $x$-axis, translate $F$ 5 units down, rotate $F$ 90° counterclockwise
   C. Translate $F$ 3 units left, translate $F$ 3 units down, rotate $F$ 90° clockwise
   D. Reflect $F$ over the $y$-axis, reflect $F$ over the $x$-axis, rotate $F$ by 180°


**Problem Solving**

**4-4 Determinants and Cramer’s Rule**

As Kristin prepares for a triathlon, she makes a chart of her exercise time, along with the calories burned each day. Part of her chart is shown in the table below. How many calories per hour does she burn for each activity?

<table>
<thead>
<tr>
<th>Triathlon Training Record</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>Friday</td>
</tr>
<tr>
<td>Saturday</td>
</tr>
<tr>
<td>Sunday</td>
</tr>
</tbody>
</table>

1. Write a system of equations that relates Kristin’s exercise time to the number of calories burned each day. Use $s$, $c$, and $r$ for the calories burned per hour for the three activities.

2. Write the coefficient matrix for the system of equations.

3. What is the value, $D$, for the determinant of the coefficient matrix?

4. Use Cramer’s rule to solve this system of equations. Give the values for $s$, $c$, and $r$.

---

Choose the letter for the best answer.

5. Ty has a bag of pennies, nickels, and dimes. He has 10 times as many pennies as dimes. He has a total of 52 coins and twice as many nickels as dimes. Which coefficient matrix could you use to solve this problem?

   A \[
   \begin{bmatrix}
   1 & 1 & 1 \\
   1 & 0 & -10 \\
   0 & 1 & -2
   \end{bmatrix}
   \]

   B \[
   \begin{bmatrix}
   1 & 1 & 1 \\
   10 & 0 & -1 \\
   0 & 2 & -1
   \end{bmatrix}
   \]

   C \[
   \begin{bmatrix}
   1 & 1 & 1 \\
   1 & 0 & -10 \\
   0 & 1 & 2
   \end{bmatrix}
   \]

   D \[
   \begin{bmatrix}
   1 & 1 & 1 \\
   10 & 0 & -1 \\
   0 & -2 & 1
   \end{bmatrix}
   \]

6. Phyllis collects silver dollars and Kennedy half-dollars. She has 5 times as many half-dollars as dollar coins. She has a total of 192 coins. Which solution could you use to find the number of silver dollars Phyllis has?

   A \[
   \begin{bmatrix}
   192 & 1 \\
   0 & -5
   \end{bmatrix}
   \]

   B \[
   \begin{bmatrix}
   1 & 192 \\
   -6 & 1
   \end{bmatrix}
   \]

   C \[
   \begin{bmatrix}
   1 & 192 \\
   -5 & 0
   \end{bmatrix}
   \]

   D \[
   \begin{bmatrix}
   192 & 1 \\
   0 & 1
   \end{bmatrix}
   \]
Problem Solving

**Matrix Inverses and Solving Systems**

For his job at a local restaurant, Alex researches and compares prices. He buys three packaged salad lunches from a competitor. He wants to find the price of one ounce of each kind of salad.

<table>
<thead>
<tr>
<th>Salad Medley</th>
<th>Tasty Threesome</th>
<th>Sampler Salad Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainbow pasta: 3 oz</td>
<td>Tender tuna: 2 oz</td>
<td>Fruit cup: 2 oz</td>
</tr>
<tr>
<td>Tasty tuna: 3 oz</td>
<td>Mixed pasta: 1 oz</td>
<td>Tuna mix: 2 oz</td>
</tr>
<tr>
<td>Fresh fruit: 1 oz</td>
<td>Seasonal fruit: 3 oz</td>
<td>Pasta medley: 2 oz</td>
</tr>
<tr>
<td>$5.52</td>
<td></td>
<td>$4.68</td>
</tr>
</tbody>
</table>

1. Write a system of equations, using $p$, $t$, and $f$ as the cost per ounce of each kind of salad.

2. Set up the matrix equation, $AX = B$.

3. Find the determinant of matrix $A$.

4. Find $A^{-1}$.

5. Solve $X = A^{-1} B$ for $X$.

6. What is the price per ounce for each kind of salad?

A pharmacist is preparing saline solutions. She has a 2% saline solution and a 12% saline solution. How much of each solution should she mix to prepare 10 liters of a 10% solution? Choose the letter for the best answer.

7. Which represents the matrix equation for this problem?

- $A \begin{bmatrix} 1 & 1 \\ 0.02 & 0.12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$
- $B \begin{bmatrix} 1 & 1 \\ 0.2 & 0.12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$
- $C \begin{bmatrix} 1 & 1 \\ 0.12 & 0.02 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$
- $D \begin{bmatrix} 1 & 1 \\ 0.02 & 0.12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$

8. Which matrix is the inverse of the coefficient matrix for this problem?

- $A \begin{bmatrix} 1 & 1.2 \\ 10 & 0.2 \end{bmatrix}$
- $B \begin{bmatrix} 1.2 & -10 \\ -0.2 & 10 \end{bmatrix}$
- $C \begin{bmatrix} -1.2 & 1 \\ 0.2 & -1 \end{bmatrix}$
- $D \begin{bmatrix} 1.2 & -10 \\ -0.2 & 1 \end{bmatrix}$
At the annual craft show, the Ceramics Club members sell mugs for $6.00, bowls for $5.50, and plates for $9.50. They have for sale one more bowl than the number of plates and 3 times as many mugs as plates. They sold everything for a total of $236.50. How many of each item did they sell?

1. Write a system of equations to represent the problem, using $m$, $b$, and $p$ for the variables.

2. Write the augmented matrix for the system of equations.

3. Use your calculator to find the reduced row-echelon form of the augmented matrix.

4. How many of each item did the Ceramics Club sell?

Students earned points for finishing first, second, and third in the field day games. Jake earned a total of 38 points, Wanda earned 33 points, and Jill earned 29 points. How many points were earned for each first-, second-, and third-place finish? Choose the letter for the best answer.

5. Which augmented matrix models the problem?

6. Which matrix in reduced row-echelon form is the solution to the problem?

\[
\begin{array}{ccc}
4 & 1 & 1; 38 \\
0 & 5 & 3 ; 33 \\
1 & 0 & 2 ; 29 \\
\end{array} \quad \begin{array}{ccc}
4 & 2 & 0 ; 38 \\
1 & 5 & 0 ; 33 \\
0 & 3 & 2 ; 29 \\
\end{array}
\]

\[
\begin{array}{ccc}
1 & 0 & 0 ; 8 \\
0 & 1 & 0 ; 5 \\
0 & 0 & 1 ; 3 \\
\end{array} \quad \begin{array}{ccc}
1 & 0 & 0 ; 10 \\
0 & 1 & 0 ; 6 \\
0 & 0 & 1 ; 3 \\
\end{array}
\]

\[
\begin{array}{ccc}
4 & 0 & 2 ; 38 \\
1 & 5 & 0 ; 33 \\
1 & 3 & 2 ; 29 \\
\end{array} \quad \begin{array}{ccc}
4 & 1 & 2 ; 38 \\
0 & 3 & 5 ; 33 \\
1 & 2 & 3 ; 29 \\
\end{array}
\]

\[
\begin{array}{ccc}
0 & 0 & 1 ; 8 \\
0 & 1 & 0 ; 5 \\
1 & 0 & 0 ; 3 \\
\end{array} \quad \begin{array}{ccc}
0 & 0 & 1 ; 10 \\
0 & 1 & 0 ; 6 \\
1 & 0 & 0 ; 3 \\
\end{array}
\]
Christa and Jelani are standing at the top of the Leaning Tower of Pisa in Italy, 185 feet above the ground. Jelani wonders what the path of a dropped object would be as it falls to the ground from the top of the tower. The height of an object after \( t \) seconds is given by the function, \( f(t) = -16t^2 + 185 \).

1. Complete the table to show the height, \( f(t) \), of the object for different values of \( t \).

2. Plot the ordered pairs from the table and draw the graph to show the path of the object.

<table>
<thead>
<tr>
<th>Time ((t))</th>
<th>( f(t) = -16t^2 + 185 )</th>
<th>((t, f(t)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( f(0) = -16(0)^2 + 185 )</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( f(1) = -16(1)^2 + 185 )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. What is the parent function for the graph? __________________________________________________________________________

4. What is the name for this U-shaped curve? __________________________________________________________________________

5. Describe the transformations of the parent function into \( f(t) = -16t^2 + 185 \), which describes the path of an object falling from 185 feet.

Choose the letter for the best answer.

6. Mario dropped a wrench from the top of a sailboat mast 58 feet high. Which function describes the path of the falling wrench?
   - A \( f(t) = 16(t - 58)^2 - 185 \)
   - B \( f(t) = -16(t - 58)^2 + 185 \)
   - C \( f(t) = 16t^2 - 58 \)
   - D \( f(t) = -16t^2 + 58 \)

7. Delle wants to transform the parent function \( f(t) = t^2 \) into \( f(t) = -4(t - 0.6)^2 + 6 \). Which is NOT a step in that transformation?
   - A Translation 6 units up
   - B Translation 0.6 unit left
   - C Reflection across the x-axis
   - D Vertical stretch by a factor of 4
Kim wants to buy a used car with good gas mileage. He knows that the miles per gallon, or mileage, varies according to various factors, including the speed. He finds that highway mileage for the make and model he wants can be approximated by the function 

\[ f(s) = -0.03s^2 + 2.4s - 30, \]

where \( s \) is the speed in miles per hour.

He wants to graph this function to estimate possible gas mileages at various speeds.

1. Determine whether the graph opens upward or downward.

2. Identify the axis of symmetry for the graph of the function.

3. Find the \( y \)-intercept.

4. Find the vertex.

5. Graph the function.

6. a. Does the curve have a maximum or a minimum value?

b. What is the value of the \( y \)-coordinate at the maximum or minimum?

c. Explain what this point means in terms of gas mileage.

A ball is hit into the air from a height of 4 feet. The function 

\[ g(t) = -16t^2 + 120t + 4 \]

can be used to model the height of the ball where \( t \) is the time in seconds after the ball is hit. Choose the letter for the best answer.

7. About how long is the ball in the air?
   A 3.5 seconds  
   B 3.75 seconds  
   C 7 seconds  
   D 7.5 seconds

8. What is the maximum height the ball reaches?
   A 108 feet  
   B 124 feet  
   C 229 feet  
   D 394 feet
Problem Solving

LESSON 5-3 Solving Quadratic Equations by Graphing and Factoring

Erin and her friends launch a rocket from ground level vertically into the air with an initial velocity of 80 feet per second. The height of the rocket, \( h(t) \), after \( t \) seconds is given by \( h(t) = -16t^2 + 80t \).

1. They want to find out how high they can expect the rocket to go and how long it will be in the air.
   a. Use the standard form \( f(x) = ax^2 + bx + c \) to find values for \( a \), \( b \), and \( c \).
   b. Use the coordinates for the vertex of the path of the rocket to find \( t \), the number of seconds the rocket will be in the air before it starts its downward path.
   c. Substitute the value for \( t \) in the given function to find the maximum height of the rocket. How high can they expect their rocket to go?
   d. Megan points out that the rocket will have a height of zero again when it returns to the ground. How long will the rocket stay in the air?

2. Megan gets ready to launch the same rocket from a platform 21 feet above the ground with the same initial velocity. How long will the rocket stay in the air this time?
   a. Write a function that represents the rocket's path for this launch.
   b. Factor the corresponding equation to find the values for \( t \) when \( h \) is zero.
   c. Erin says that the roots of the equation are \( t = 5.25 \) and \( t = -20.25 \) and that the rocket will stay in the air 5.5 seconds. Megan says she is wrong. Who is correct? How do you know?

Choose the letter for the best answer.

3. Which function models the path of a rocket that lands 3 seconds after launch?
   A \( h(t) = -16t^2 + 32t + 48 \)
   B \( h(t) = -16t^2 + 32t + 10.5 \)
   C \( h(t) = -16t^2 + 40t + 48 \)
   D \( h(t) = -16t^2 + 40t + 10.5 \)

4. Megan reads about a rocket whose path can be modeled by the function \( h(t) = -16t^2 + 100t + 15 \). Which could be the initial velocity and launch height?
   A 15 ft/s; 100 ft off the ground
   B 16 ft/s; 100 ft off the ground
   C 100 ft/s; 15 ft off the ground
   D 171 ft/s; 15 ft off the ground
Problem Solving

5-4 Completing the Square

Sean and Mason run out of gas while fishing from their boat in the bay. They set off an emergency flare with an initial vertical velocity of 30 meters per second. The height of the flare in meters can be modeled by \( h(t) = -5t^2 + 30t \), where \( t \) represents the number of seconds after launch.

1. Sean thinks the flare should reach at least 15 meters to be seen from the shore. They want to know how long the flare will take to reach this height.

   a. Write an equation to determine how long it will take the flare to reach 15 meters.

   \[ 15 = -5t^2 + 30t \]

   b. Simplify the function so you can complete the square.

   \[ t^2 - 6t + \frac{9}{4} = \frac{45}{4} \]

   c. Solve the equation by completing the square.

   \[ t = \frac{3}{2} \text{ or } t = 3 \]

   d. Mason thinks that the flare will reach 15 meters in 5.4 seconds. Is he correct? Explain.

   Possible answer: He is partially correct. The flare will first reach 15 meters at 0.6 second after firing and then again at 5.4 seconds. (The function has two solutions.)

   e. Sean thinks the flare will reach 15 meters sooner, but then the flare will stay above 15 meters for about 5 seconds. Is he correct? Explain.

   Possible answer: He is correct. The flare will first reach 15 meters at 0.6 second after firing. Also, the difference between 5.4 and 0.6 seconds (the two solutions) is 4.8 seconds, which is about 5 seconds.

2. Sean wants to know how high the flare will reach above the surface of the water.

   a. Write the function in vertex form, factoring so the coefficient of \( t^2 \) is 1.

   \[ h(t) = -5(t - 3)^2 + 45 \]

   b. Complete the square using the vertex form of the function.

   \[ h(t) = -5(t - 3)^2 + 45 \]

   c. How high will the flare reach?

   \[ 45 \]

Choose the letter for the best answer.

3. Use the vertex form of the function to determine how long after firing the flare it will reach its maximum height.

   A 3 s
   B 5 s
   C 9 s
   D 15 s

4. The boys fire a similar flare from the deck 5 meters above the water level. Which statement is correct?

   A The flare will reach 45 m in 3 s.
   B The flare will reach 50 m in 3 s.
   C The flare will reach 45 m in 3.5 s.
   D The flare will reach 50 m in 3.5 s.
Problem Solving

5-5 Complex Numbers and Roots

At a carnival, a new attraction allows contestants to jump off a springboard onto a platform to be launched vertically into the air. The object is to ring a bell located 20 feet overhead. The distance from the bell in feet is modeled by the function \( dt = 16t^2 - bt + 20 \), where \( t \) is the time in seconds after leaving the platform, and \( b \) is the takeoff velocity from the platform.

1. Kate watches some of the contestants. She theorizes that if the platform launches a contestant with a takeoff velocity of at least 32 feet per second, the contestant can ring the bell.
   a. Find the zeros for the function using 32 feet per second as the takeoff velocity.
   b. Is Kate's theory valid? Explain.

2. Mirko suggests they vary the value of \( b \) and determine for which values of \( b \) the roots are real.
   a. Complete the table to show the roots for different values of \( b \).
   b. For which values of \( b \) in the table are the roots real?

<table>
<thead>
<tr>
<th>( b )</th>
<th>Function</th>
<th>Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>( d(t) = 16t^2 - 24t + 20 )</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>( d(t) = 16t^2 - _t + 20 )</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>( d(t) = 16t^2 - _t + 20 )</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>( d(t) = 16t^2 - _t + 20 )</td>
<td></td>
</tr>
</tbody>
</table>

   c. What difference does it make if the roots are real?

3. Using the results from the table, and the function, estimate the minimum takeoff velocity needed for a contestant to be able to ring the bell.

Choose the letter for the best answer.

4. Mirko suggests using four bells at heights of 15, 20, 25, and 30 feet from the platform. How many of the bells can a contestant reach if the takeoff velocity is 32 feet per second?
   A 3  C 1
   B 2  D 0

5. At what height must a bell be placed for a contestant to reach it with a takeoff velocity of 48 feet per second?
   A 20 feet or less
   B 25 feet or less
   C 30 feet or less
   D 36 feet or less
Problem Solving
The Quadratic Formula

In a shot-put event, Jenna tosses her last shot from a position of about 6 feet above the ground with an initial vertical and horizontal velocity of 20 feet per second. The height of the shot is modeled by the function \( h(t) = -16t^2 + 20t + 6 \), where \( t \) is the time in seconds after the toss. The horizontal distance traveled after \( t \) seconds is modeled by \( d(t) = 20t \).

1. Jenna wants to know the exact distance the shot travels at a velocity of 20 feet per second.
   a. Use the Quadratic Formula \( t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) to solve the height function for \( t \).
   b. Use the value for \( t \) and the distance function to find the distance her shot travels.

2. Jenna is working to improve her performance. She makes a table to show how the horizontal distance varies with velocity. Complete the table.

<table>
<thead>
<tr>
<th>Velocity (ft/s)</th>
<th>Formula</th>
<th>Time (s)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 22</td>
<td>( t = \frac{-22 \pm \sqrt{(22)^2 - 4(-16)(6)}}{2(-16)} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. 25</td>
<td>( t = \frac{-25 \pm \sqrt{(25)^2 - 4(-16)(6)}}{2(-16)} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 28</td>
<td>( t = \frac{-28 \pm \sqrt{(28)^2 - 4(-16)(6)}}{2(-16)} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Jenna has not reached her full potential yet. Her goal is to toss the shot from a height of 6 feet 6 inches with a vertical and horizontal velocity of 30 feet per second. Choose the letter for the best answer.

3. If she achieves her goal, how long will her shot stay in the air?
   A. 1.65 s
   B. 1.87 s
   C. 2.07 s
   D. 2.27 s

4. If she achieves her goal, what horizontal distance will the shot travel?
   A. 41.4 ft
   B. 56.1 ft
   C. 62.1 ft
   D. 68.1 ft
Problem Solving
Solving Quadratic Inequalities

The manager at Travel Tours is proposing a fall tour to Australia and New Zealand. He works out the details and finds that the profit \( P \) for \( x \) persons is \( P(x) = -28x^2 + 1400x - 3496 \). The owner of Travel Tours has decided that the tour will be canceled if the profit is less than $10,000.

1. a. Write an inequality that you could use to find the number of people needed to make the tour possible.

\[ -28x^2 + 1400x - 3496 < 10000 \]

b. Solve the related equation to find the critical values.

\[ -28x^2 + 1400x - 3496 = 10000 \]
\[ -28x^2 + 1400x - 13496 = 0 \]
\[ x = 13.04, 36.96 \]

c. Test an \( x \)-value in each interval.

<table>
<thead>
<tr>
<th>( x )-value</th>
<th>Evaluate</th>
<th>( P \geq 10000? )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>(-28(10)^2 + 1400(10) - 3496)</td>
<td>No</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. How many people will Travel Tours need to make the tour possible?

From 14 to 36 people

2. A year later, the owner of Travel Tours decides that the Australia/New Zealand tour will have to make a profit of at least $12,000 for the tour to be possible. What effect will this have on the range of people able to take this tour?

Possible answer: The range is narrower. There must be between 17 and 33 people to take the tour.

The manager plans a tour to the Fiji Islands and determines that the profit \( P \) for \( x \) persons is \( P(x) = -40x^2 + 1920x - 3200 \). Choose the letter for the best answer.

3. In order to make $10,000 profit, how many people will it take for this tour to happen?

A Between 9 and 39 people
B Between 14 and 36 people
C At least 22 people
D At least 30 people

4. The owner thinks the company should make at least $15,000 profit on the Fiji Islands tour. How many people will it take for the tour to happen?

A Between 9 and 39 people
B Between 13 and 35 people
C At least 22 people
D At least 35 people
**Problem Solving**

**5-8 Curve Fitting with Quadratic Models**

Ellen and Kelly test Ellen’s new car in an empty parking lot. They mark a braking line where Ellen applies the brakes. Kelly then measures the distance from that line to the place where Ellen stops, for speeds from 5 miles per hour to 25 miles per hour.

<table>
<thead>
<tr>
<th>Brake Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (mi/h)</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>Stopping Distance (ft)</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>46</td>
</tr>
<tr>
<td>65</td>
</tr>
</tbody>
</table>

1. Ellen wants to know the stopping distance at 60 miles per hour. She cannot drive the car at this speed in the parking lot, so they decide to try curve fitting, using the data they have collected.

   a. Can you use a quadratic function to represent the data in the table? Explain how you know.

   b. Use three points to write a system of equations to find $a$, $b$, and $c$ in $f(x) = ax^2 + bx + c$.

   c. Use any method to solve 3 equations with 3 variables. Find the values for $a$, $b$, and $c$.

   d. Write the quadratic function that models the stopping distance of Ellen’s car.

   e. What is the stopping distance of Ellen’s car at 60 miles per hour?

The table shows the sizes and prices of decorative square patio tiles. Choose the letter for the best answer.

<table>
<thead>
<tr>
<th>Patio Tiles Sale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side Length (in.)</td>
</tr>
<tr>
<td>Price Each ($)</td>
</tr>
</tbody>
</table>

2. What quadratic function models the price of the patio tiles?

   A $P(x) = 0.4x^2$
   B $P(x) = 0.04x^2$
   C $P(x) = 0.04x^2 + 0.4x$
   D $P(x) = 0.04x^2 + x + 0.4$

3. What is the second difference constant for the data in the table?

   A 1.44
   B 1.08
   C 0.72
   D 0.36
Problem Solving

Operations with Complex Numbers

Hannah and Aoki are designing fractals. Aoki recalls that many fractals are based on the Julia Set, whose formula is

\[ Z_{n+1} = (Z_n)^2 + c \]

where \( c \) is a constant. Hannah suggests they make their own fractal pattern using this formula, where \( c = 1 \) and \( Z_1 = 1 + 2i \).

1. Complete the table to show values of \( n \) and \( Z_n \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( Z_{n+1} = (Z_n)^2 + c )</th>
<th>( Z_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( Z_1 = 1 + 2i )</td>
<td>( Z_1 = 1 + 2i )</td>
</tr>
<tr>
<td>2</td>
<td>( Z_2 = (1 + 2i)^2 + 1 )</td>
<td>( Z_2 = )</td>
</tr>
<tr>
<td>3</td>
<td>( Z_3 = \left( _ _ \left/ _ _ \right. \right)^2 + 1 )</td>
<td>( Z_3 = )</td>
</tr>
<tr>
<td>4</td>
<td>( Z_4 = \left( _ _ \left/ _ _ \right. \right)^2 + 1 )</td>
<td>( Z_4 = )</td>
</tr>
</tbody>
</table>

2. Four points are shown on the complex plane. Which point is not part of the fractal pattern they have created? Explain.

Choose the letter for the best answer.

3. Aoki creates a second pattern by changing the value of \( c \) to 3. What happens to \( Z_n \) as \( n \) increases?
   - A The imaginary part is always twice the real part.
   - B The real and imaginary parts become equal.
   - C The real part becomes zero.
   - D The imaginary part becomes zero.

4. Hannah changes the formula to

\[ Z_{n+1} = \frac{1}{(Z_n)^2} + c \]

Leaving \( c = 1 \) and \( Z_1 = 1 + 2i \), what is the value of \( Z_2 \)?
   - A 0.48 – 0.16i
   - B 0.88 – 0.16i
   - C 1.2 – 0.4i
   - D 2.2 – 0.4i

5. Aoki takes Hannah’s new formula, leaves \( c = 1 \), and sets \( Z_1 = \frac{1}{1 + 2i} \).

What is the value of \( Z_3 \)?
   - A \( Z_3 = 0.97 + 0.04i \)
   - B \( Z_3 = 2 + 2i \)
   - C \( Z_3 = 0.48 – 0.16i \)
   - D \( Z_3 = 147.4 + i \)

6. Hannah reverts to

\[ Z_{n+1} = (Z_n)^2 + c \]

She sets \( Z_1 = i \) and \( c = i \). Which statement is NOT true?
   - A \( Z_n \) flip-flops between \((-1 + i)\) and \((-i)\).
   - B The coefficient of \( i \) never reaches 2.
   - C The imaginary part becomes zero.
   - D On a graph \( Z_1 – Z_3 \) create a triangle.
Problem Solving

Polynomials

As part of a project to build a model castle, Julian wants to find the surface area of solid towers of various sizes, shaped like the one shown in the figure below. The diameter of the circular base is \(d\) inches, the height of the cylinder is \(d + 4\) inches, and the slant height of the right circular cone is \(d - 0.6\) inch.

1. The general formula for the surface area of a cone is \(SA = \pi r^2 + \pi rs\), where \(r\) is the radius of the base, and \(s\) is the slant height of the cone.
   a. Write the formula in terms of \(d\).
   b. What part of the formula will you use to find the surface area of the cone part of the model? Why?

2. The general formula for the surface area of a cylinder (with radius \(r\) and height \(h\)) is \(SA = 2\pi r^2 + 2\pi rh\).
   a. Write the formula in terms of \(d\).
   b. What part of the formula will you use to find the surface area of the cylinder part of the model? Why?

3. Write a general polynomial expression for the surface area of the model tower.

Choose the letter for the best answer.

4. What is the approximate surface area in square inches of a tower with a diameter of 5 inches?
   - A 278
   - B 196
   - C 44
   - D 38

5. What is the approximate surface area in square inches of a tower with a diameter of 10 inches?
   - A 176
   - B 278
   - C 666
   - D 1174

6. What is the approximate surface area in square inches of a tower where the height of the cylinder is 12 inches?
   - A 931
   - B 716
   - C 445
   - D 395

7. What is the approximate surface area in square inches of a tower where the slant height of the cone is 3.4 inches?
   - A 103
   - B 134
   - C 158
   - D 268
Problem Solving

Multiplying Polynomials

Latesha is making an open wooden toy box to hold the building blocks at her day care center. She has a square panel of cedar with side length of 24 inches. The first step is to cut out congruent squares from each corner. She needs to know what the side length of the cutout square should be in order for the finished toy box to have the greatest volume possible.

1. Draw a sketch to help solve the problem.

2. The toy box will be square and \( x \) inches deep. Write an expression for the side length of the finished box.

3. Write an equation to represent the volume.

4. Express the volume as the sum of monomials.

5. Latesha decides to try some possible values for \( x \). She knows that \( x \) must be less than 12. Explain why.

6. Complete the table for each value of \( x \).

<table>
<thead>
<tr>
<th>( x ) (in.)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (sq in.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Latesha decides that she will use an integer value for \( x \), so that she does not have to cut fractions of an inch.

   a. What value for \( x \) should she choose?
   
   b. Explain why this is the best choice.

   c. What are the dimensions of her finished toy box?
Problem Solving

Dividing Polynomials

An art class is making pedestals in the shape of regular prisms to display sculptures in an art show. Blake is in charge of the mirrors for the tops of the pedestals. He needs to estimate the total area of the mirrored surfaces. He will use that total to help determine the amount of mirrored product to purchase.

The figures below show the shape of the bases for each of the three kinds of prisms that will be used for pedestals. Each regular polygon has a side length of \( x \). Recall that, for a prism, \( V = Bh \).

1. The triangular prism has a height of \( 2x + 1 \) and its volume can be modeled by \( V(x) = \frac{\sqrt{3}}{2} x^3 + \frac{\sqrt{3}}{4} x^2 \). What is the area of the top of the pedestal?

Choose the letter for the best answer.

2. The volume of the pentagonal prism can be modeled by \( V = 6.88x^3 - 1.72x^2 \). Which expression represents the area of the top of the prism if the height is \( 4x - 1 \)?
   A. \( 0.57x^2 \)
   B. \( 1.72x^2 \)
   C. \( 2.28x^2 \)
   D. \( 6.88x^2 \)

3. The volume of the octagonal prism can be modeled by \( V = 4.83x^3 - 24.15x^2 \). Which expression represents the area of the top of the prism if the height is \( x - 5 \)?
   A. \( 48.3x^2 \)
   B. \( 38.64x^2 \)
   C. \( 4.83x^2 \)
   D. \( 3.86x^2 \)

4. Which expression represents the total area that will be mirrored?
   A. \( A = x^2\left(\frac{\sqrt{3}}{4} + 6.55\right) \)
   B. \( A = 6.98x \)
   C. \( A = 12.58x^3 + 22.86x^2 \)
   D. \( A = \sqrt{6.98}x \)

5. If \( x = 5 \), what is the total mirrored area in square units?
   A. \( 6.98 \)
   B. \( 34.9 \)
   C. \( 69.8 \)
   D. \( 174.5 \)
Problem Solving

6-4
Factoring Polynomials

Paulo is drawing plans for a set of three proportional nesting baskets, in the shape of open rectangular prisms.

1. The volume for the middle-sized basket (B) can be modeled by the function \( V_B(x) = x^3 - 8x^2 + 4x + 48 \).
   Use the graph to factor \( V_B \).
   a. What are the values of \( x \) where \( V_B = 0 \)?
   
   
   b. Use these zeros to write the factors.

2. The volume for the largest basket (C) can be modeled by the function \( V_C(x) = 2x^3 + 10x^2 + 8x \).
   Use the graph to factor \( V_C \).
   a. What are the values of \( x \) where \( V_C = 0 \)?
   
   
   b. Use these zeros to write the factors.

3. The volume for the smallest basket (A) can be modeled by the function \( V_A(x) = x^3 - 22x^2 + 157x - 360 \).
   Use the graph to factor \( V_A \).
   a. What are the values of \( x \) where \( V_A = 0 \)?
   
   
   b. Use these zeros to write the factors.

4. Complete the table. Use \( x = 12 \) units to find the actual dimensions and volume.

<table>
<thead>
<tr>
<th>Basket</th>
<th>Dimensions (in terms of ( x ))</th>
<th>Actual Dimensions</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Are the actual dimensions of the three baskets proportional? Explain.

6. Are the volumes of the three baskets proportional? Explain.
Most airlines have rules concerning the size of checked baggage. The rules for Budget Airline are such that the dimensions of the largest bag cannot exceed 45 in. by 55 in. by 62 in. A designer is drawing plans for a piece of luggage that athletes can use to carry their equipment. It will have a volume of 76,725 cubic inches. The length is 10 in. greater than the width and the depth is 14 in. less than the width. What are the dimensions of this piece of luggage?

1. Write an equation in factored form to model the volume of the piece of luggage.

2. Multiply and set the equation equal to zero.

3. Think about possible roots of the equation. Could a root be a multiple of 4? ___________ a multiple of 5? ___________
a multiple of 10? ___________. How do you know?

4. Use synthetic substitution to test possible roots. Choose positive integers that are factors of the constant term and reasonable in the context of the problem.

<table>
<thead>
<tr>
<th>Possible Root</th>
<th>1</th>
<th>-4</th>
<th>-140</th>
<th>-76,725</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Choose the letter for the best answer.

5. Which equation represents the factored polynomial?
   A \((w + 55)(w^2 + 25w + 1550) = 0\)
   B \((w - 35)(w^2 + 60w + 1405) = 0\)
   C \((w - 45)(w^2 + 41w + 1705) = 0\)
   D \((w - 4)(w^2 - 140w + 76,725) = 0\)

6. Which could be the dimensions of this piece of luggage?
   A 31 in. by 45 in. by 55 in.
   B 45 in. by 55 in. by 55 in.
   C 45 in. by 45 in. by 55 in.
   D 45 in. by 55 in. by 62 in.
Problem Solving

**Fundamental Theorem of Algebra**

A company that makes accessories for cars needs a container like that shown at the right to hold touch-up paint. The hemispherical top will be fitted with a brush applicator. The cylindrical part of the container should be 4 inches tall. The volume of the entire container is \( \frac{13}{12} \pi \) cubic inches. Find the value of \( x \), the radius of the hemisphere.

1. **a.** Write a formula for the volume of the cylindrical part of the container.

   \[ V_{cylinder} = \pi r^2 h \]

   \[ V_{cylinder} = \pi x^2 (4) \]

2. **b.** Write a formula for the volume of the hemispherical part of the container.

   \[ V_{hemisphere} = \frac{2}{3} \pi r^3 \]

   \[ V_{hemisphere} = \frac{2}{3} \pi x^3 \]

3. Write an equation to represent the total volume of the container.

   \[ V_{total} = V_{cylinder} + V_{hemisphere} \]

   \[ \frac{13}{12} \pi = 4 \pi x^2 + \frac{2}{3} \pi x^3 \]

4. **Write the equation in standard form.**

   \[ \frac{2}{3} \pi x^3 + 4 \pi x^2 - \frac{13}{12} \pi = 0 \]

5. **Graph the equation with a graphing calculator.**

   Hint: Use a window with \( x \)-values from -8 to 5 with a scale of 1, and \( y \)-values from -20 to 250 with a scale of 30 to see the general shape of the graph. Sketch the graph. Then focus on the area of the positive root by using a window of -8 to 3 on the \( x \)-axis and -20 to 20 on the \( y \)-axis. Use Trace to help you find a possible positive root.

5. **Verify the root using synthetic substitution.**

   What is the positive root?

6. **Use the Quadratic Formula to find approximate values for the other two roots.**

   Explain why these two roots cannot also be solutions to the problem.

7. **What is the value of \( x \), the radius of the hemisphere, for this paint container?**
Problem Solving

Investigating Graphs of Polynomial Functions

The Spanish Club members are baking and selling fruit bars to raise money for a trip. They are going to make open boxes to display the bars from sheets of cardboard that are 11 inches by 17 inches. They will cut a square from each corner and fold up the sides and tape them. Find the maximum value for the volume of the box and find its dimensions.

1. Write a formula to represent the volume of the box.

2. a. Write the equation in standard form.
   b. Is the leading coefficient positive or negative?
   c. Is the degree of the polynomial even or odd?
   d. Describe the end behavior of the graph.

3. Use a graphing calculator to graph the equation. Hint: Try a window from $-10$ to $10$ on the $x$-axis, with a scale of $1$, and from $-500$ to $500$ on the $y$-axis, with a scale of $100$.
   a. How many turning points does the graph have?
   b. Estimate the local maxima and minima from the graph.

4. What values of $x$ are excluded as solutions because they do not make sense for this problem?

5. Use the CALC menu on your graphing calculator to find the approximate values of $x$ and $y$ at the local maximum for the graph.

6. What is the maximum volume of the box?

7. What are the dimensions of the box to the nearest tenth of an inch?

Choose the letter for the best answer.

8. Arturo is going to build a dog run using one side of his house and 100 feet of fencing. His design has an area that can be modeled by $A(x) = 100x - 7x^2$. What is the maximum area he can enclose?
   A $357 \text{ ft}^2$
   B $204 \text{ ft}^2$
   C $100 \text{ ft}^2$
   D $70 \text{ ft}^2$

9. In order to eliminate some choices on a standardized test, Ruth identifies which of these functions could NOT have a local maximum.
   A $f(x) = -7x^2 + 5x + 2$
   B $f(x) = -7x^3 + 5x - 11$
   C $f(x) = 7x^3 - 5x^2 - 2$
   D $f(x) = 7x^2 - 3x - 18$
Problem Solving

Transforming Polynomial Functions

A traffic engineer determines that the number of cars passing through a certain intersection each week can be modeled by 

\[ C(x) = 0.02x^3 + 0.4x^2 + 0.2x + 35, \]
where \( x \) is the number of weeks since the survey began. A new road has just opened that affects the traffic at that intersection. Let \( N(x) = C(x) + 200. \)

1. Find the rule for \( N(x) \).

2. What transformation of \( C(x) \) is represented by \( N(x) \)?

3. On the graph of \( C(x) \), sketch the graph for \( N(x) \).

4. Use a graphing calculator to graph \( N(x) \). Use a window from 0 to 20 with a scale of 1 on the \( x \)-axis and from 0 to 500 with a scale of 1 on the \( y \)-axis. Compare it to your sketch. Explain why only the values in Quadrant 1 are considered for this problem.

5. Explain the meaning of the transformation of \( C(x) \) into \( N(x) \) in terms of the weekly number of cars passing through the intersection.

6. Emergency roadwork temporarily closes off most of the traffic to this intersection. Write a function \( R(x) \) that could model the effect on \( C(x) \). Explain how the graph of \( C(x) \) might be transformed into \( R(x) \).

7. Describe the transformation \( 2C(x) \) by writing the new rule and explaining the change in the context of the problem.
Problem Solving

6-9 Curve Fitting with Polynomial Models

Carla has been making a “wild scape” in her backyard. The table shows the number of birds visiting her feeder at the same hour on the first day of each month since she began her project. Use a polynomial model to make a reasonable estimate of the number of birds there might be in July.

| Birds at Feeder from 7:00 to 8:00 A.M. |
|------|------|------|------|------|------|
| Jan  | Feb  | Mar  | Apr  | May  | Jun  |
| 3    | 8    | 18   | 36   | 65   | 108  |

1. Use finite differences to determine the degree of the polynomial that best fits the data.
   a. First differences
   b. Second differences
   c. Third differences
   d. Fourth differences
   e. Which degree polynomial best describes the data?

2. Use your graphing calculator to find values for $R^2$.
   a. For LinReg, $R^2 =$
   b. For QuadReg, $R^2 =$
   c. For CubicReg, $R^2 =$

3. Write the polynomial model for this data.

4. Use your polynomial model to make a reasonable estimate of the number of birds there might be in July.

The table below shows the number of travel insurance policies sold by a travel agency over a six-year period. Choose the letter for the best answer.

<table>
<thead>
<tr>
<th>Year</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Policies Sold</td>
<td>73</td>
<td>126</td>
<td>163</td>
<td>185</td>
<td>192</td>
<td>184</td>
</tr>
</tbody>
</table>

5. Which function best models the data?
   A $f(x) = -0.926x^2 - 6.88x + 6.3$
   B $f(x) = -7.59x^2 + 75.27x + 5.5$
   C $f(x) = 0.05x^3 + 8.08x^2 + 76.74x + 4.3$
   D $f(x) = -0.02x^3 + 0.34x^2 - 9.47x + 2.8$

6. Use the polynomial model to estimate the number of policies that may be sold in 2008.
   A About 150
   B About 140
   C About 130
   D About 120
Problem Solving

Exponential Functions, Growth, and Decay

Justin drove his pickup truck about 22,000 miles in 2004. He read that in 1988 the average residential vehicle traveled about 10,200 miles, which increased by about 2.9% per year through 2004.

1. Write a function for the average mileage, \( m(t) \), as a function of \( t \), the time in years since 1988.

2. Assume that the 2.9% increase is valid through 2008 and use your function to complete the table to show the average annual miles driven.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>0</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m(t) )</td>
<td>10,200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Did Justin drive more or fewer miles than the average residential vehicle driver in 2004? by how much (to the nearest 100 miles)?

4. Later Justin read that the annual mileage for light trucks increased by 7.8% per year from 1988 to 2004.
   a. Write a function for the average miles driven for a light truck, \( n(t) \), as a function of \( t \), the time in years since 1988. He assumes that the average number of miles driven in 1988 was 10,200.
   b. Graph the function. Then use your graph to estimate the average number of miles driven (to the nearest 1000) for a light truck in 2004.
   c. Did Justin drive more or fewer miles than the average light truck driver in 2004? by how much?

Justin bought his truck new for $32,000. Its value decreases 9.0% each year. Choose the letter for the best answer.

5. Which function represents the yearly value of Justin’s truck?
   A \( f(t) = 32,000(1 + 0.9)^t \)
   B \( f(t) = 32,000(1 - 0.9)^t \)
   C \( f(t) = 32,000(1 + 0.09)^t \)
   D \( f(t) = 32,000(1 - 0.09)^t \)

6. When will the value of Justin’s truck fall below half of what he paid for it?
   F In 6 years
   G In 8 years
   H In 10 years
   J In 12 years
Problem Solving

Inverses of Relations and Functions

Sally and Janelle pay a total of $47.96 to camp for three nights at a state park. This includes a one-time park entrance fee of $5 and 9% sales tax. They paid $12 per night to stay for three nights last year, and the one-time park entrance fee was $5.

1. By how much per night has the price changed since last year?
   a. Write an equation for the total price, \( p \), as a function of the price per night, \( n \).
   b. Find the inverse function that models the price per night as a function of the total price.
   c. Evaluate the inverse function to find \( n \), the price per night.
   d. By how much has the price per night changed since last year?

2. Sally is thinking about whether they want to stay at the park next year. Assume that the entrance fee and the sales tax rate will not change.
   a. If the price per night does not increase from this year’s price, how much will it cost to stay for five nights next year?
   b. If the park management quotes them a price of $87.20 for five nights next year, what is the increase in the price per night?

Choose the letter for the best answer.

3. If Sally and Janelle decide that they want to spend five nights at this same park in the future and spend no more than $100, what is the maximum price per night that they can pay?
   A $16.00
   B $16.50
   C $17.00
   D $17.50

4. If the price of a camping vacation can be expressed as a function of the number of nights, what does the inverse function represent?
   F Number of nights as a function of the price per night
   G Number of nights as a function of the price of the vacation
   H Price of the vacation as a function of the price per night
   J Price of the vacation as a function of the number of nights
1. Find the acidity of rainwater in eastern Ohio.
   a. Substitute the hydrogen ion concentration for rainwater in eastern Ohio in the function for pH.
   b. Evaluate the function. What is the acidity of rainwater in eastern Ohio to the nearest tenth of a unit?

2. Find how the acidity of rainwater in central California compares to the acidity of rainwater in eastern Ohio.
   a. Write a function for the acidity of rainwater in central California.
   b. Evaluate the function. What is the acidity of rainwater in central California to the nearest tenth of a unit?
   c. Compare the pH of rainwater in the two locations. Is the pH of rainwater in eastern Ohio greater than or less than that in central California? By how much?

Choose the letter for the best answer.
3. What is the pH of rainwater in eastern Texas?
   A pH = 3.7      C pH = 4.4
   B pH = 4.0      D pH = 4.7

4. Nick makes his own vegetable juice. It has a hydrogen ion concentration of $5.9 \times 10^{-6}$ moles per liter. What is the pH of his vegetable juice?
   F pH = 4.9      H pH = 5.1
   G pH = 5.0      J pH = 5.2

5. What is the pH of a sample of irrigation water with a hydrogen ion concentration of $8.3 \times 10^{-7}$ moles per liter?
   A pH = 6.1      C pH = 6.3
   B pH = 6.2      D pH = 6.4

6. What is the pH of a shampoo sample with a hydrogen ion concentration of $1.7 \times 10^{-8}$ moles per liter?
   F pH = 7.4      H pH = 7.8
   G pH = 7.6      J pH = 8.0
Trina and Willow are researching information on earthquakes. One of the largest earthquakes in the United States, centered at San Francisco, occurred in 1906 and registered 7.8 on the Richter scale. The Richter magnitude of an earthquake, \( M \), is related to the energy released in ergs, \( E \), by the formula \( M = \frac{2}{3} \log \left( \frac{E}{10^{11.8}} \right) \).

1. Find the amount of energy released by the earthquake in 1906.
   a. Substitute 7.8 for magnitude, \( M \), in the equation.
   b. Solve for the value of \( \log E \).
   c. Willow says that \( E \) is equal to 10 to the power of the value of \( \log E \). Is she correct? What property or definition can be used to find the value of \( E \)? Explain.
   d. Trina says the energy of the 1906 earthquake was \( 3.16 \times 10^{23} \) ergs. Willow says the energy was \( 10^{23.5} \) ergs. Who is correct? How do you know?

Choose the letter for the best answer.

2. An earthquake in 1811 in Missouri measured 8.1 on the Richter scale. About how many times as much energy was released by this earthquake as by the California earthquake of 1906?
   A. 2.8
   B. 3.0
   C. 3.6
   D. 5.7

3. Another large earthquake in California measured 7.9 on the Richter scale. Which statement is true?
   F. 0.1 times as much energy was released by the larger earthquake.
   G. The difference in energy released is \( 1.3 \times 10^{23} \) ergs.
   H. The energy released by the second earthquake was \( 3.26 \times 10^{23} \) ergs.
   J. The total energy released by the two earthquakes is equal to the energy released by an 8.0 earthquake.

4. Larry wrote the following:
   \( \log_{10} 0.0038 = 3.8 \times 10^{-3} \). Which property of logarithms did he use?
   A. Product Property
   B. Quotient Property
   C. Inverse Property
   D. Power Property

5. Vijay wants to change \( \log_5 7 \) to base 10. Which expression should he use?
   F. \( \frac{\log_{10} 7}{\log_{10} 5} \)
   H. \( \frac{\log_{10} 7}{\log_5 5} \)
   G. \( \frac{\log_{10} 5}{\log_{10} 7} \)
   J. \( \frac{\log_5 5}{\log_{10} 7} \)
While John and Cody play their favorite video game, John drinks 4 cups of coffee and a cola, and Cody drinks 2 cups of brewed tea and a cup of iced tea. John recalls reading that up to 300 mg of caffeine is considered a moderate level of consumption per day. The rate at which caffeine is eliminated from the bloodstream is about 15% per hour.

1. John wants to know how long it will take for the caffeine in his bloodstream to drop to a moderate level.
   a. How much caffeine did John consume?
   b. Write an equation showing the amount of caffeine in the bloodstream as a function of time.
   c. How long, to the nearest tenth of an hour, will it take for the caffeine in John's system to reach a moderate level?

2. a. Cody thinks that it will take at least 8 hours for the level of caffeine in John's system to drop to the same level of caffeine that Cody consumed. Explain how he can use his graphing calculator to prove that.
   b. What equations did Cody enter into his calculator?
   c. Sketch the resulting graph.

Choose the letter for the best answer.

3. About how long would it take for the level of caffeine in Cody’s system to drop by a factor of 2?
   A 0.2 hour
   B 1.6 hours
   C 2.7 hours
   D 4.3 hours

4. If John drank 6 cups of coffee and a cola, about how long would it take for the level of caffeine in his system to drop to a moderate level?
   F 0.5 hour
   G 1.6 hours
   H 4.7 hours
   J 5.3 hours

Caffeine Content of Some Beverages

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Caffeine (mg per serving)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brewed coffee</td>
<td>103</td>
</tr>
<tr>
<td>Brewed tea</td>
<td>36</td>
</tr>
<tr>
<td>Iced tea</td>
<td>30</td>
</tr>
<tr>
<td>Cola</td>
<td>25</td>
</tr>
</tbody>
</table>
Problem Solving

The Natural Base, e

Irene reads that the 2004 census of whooping cranes tallied 213 birds at one wildlife refuge in Texas. This number exceeded the 2003 record by 19. If the population of whooping cranes can be modeled using the exponential growth function \( P_t = P_0 e^{kt} \), the population, \( P_t \), at time \( t \) can be found, where \( P_0 \) is the initial population and \( k \) is the growth factor. Predict the population of whooping cranes over the next few years.

1. What was the size of the population of whooping cranes in 2003?

2. Use the population figures for 2003 and 2004 to find the growth factor, \( k \).

3. Complete the table to predict the population of whooping cranes through 2010.

<table>
<thead>
<tr>
<th>Year</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population, ( P_t )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Choose the letter for the best answer.

4. Irene wants to know when the population of whooping cranes will exceed 1000. Using the 2003 population as \( P_0 \), which year is the best prediction?
   A 2017
   B 2019
   C 2021
   D 2023

5. Irene wonders how the 2010 whooping crane population would change if the growth factor doubled. Which statement is true?
   F The population would increase by a factor of \( e^2 \).
   G The population would increase by a factor of \( e^{0.0934} \).
   H The population would increase by a factor of \( e^{0.0934 \cdot (7)} \).
   J The population would increase by a factor of \( 7e^2 \).

6. How long will it take for an investment in an account paying 6% compounded continuously to double?
   A 10.2 years
   B 10.8 years
   C 11.6 years
   D 12.4 years

7. Darlene has a sample of a fossil that has 33% of its original carbon-14. Carbon-14 has a half-life of 5730 years. The decay constant for carbon-14 is \( 1.2 \times 10^{-4} \). Find the age of the fossil.
   F About 7820 years
   G About 8450 years
   H About 8980 years
   J About 9240 years
LESSON 7-7

Problem Solving

Transforming Exponential and Logarithmic Functions

Alex is studying a new species of hybrid plant. The average height of the plant can be modeled by the function \( h(t) = 2 \ln (t + 1.25) \), where \( h \) is the height in feet and \( t \) is the number of weeks after planting.

1. Alex graphs the function to see the rate an average plant grows.
   a. About how tall can he expect the plant to be after 3 weeks?

2. Alex plants seeds and finds that the height is now modeled by the parent function.
   a. Give the parent function \( g(t) \).
   b. Describe how the function \( h(t) \) is transformed from the parent function.
   c. Choose the letter of the graph that represents the parent function.

Alex experiments with different fertilizers and finds that he can change the growth curve of the hybrid plant. Choose the letter for the best answer.

3. Alex finds that the height of the plants can now be modeled by the function \( f(t) = 1.5 \ln (t + 1) + 0.4 \). Which statement describes the transformation from the parent function?
   A Translation 0.4 unit up and 1 unit left; vertical stretch by 1.5
   B Translation 1 unit up and 0.4 unit right; vertical stretch by 1.5
   C Translation 0.4 unit down and 1 unit left; horizontal compression by 1.5
   D Translation 1 unit down and 0.4 unit right; horizontal compression by 1.5

4. Alex looks at the graph of the growth of his plants after trying a different fertilizer. The graph is transformed from the parent function by a vertical compression by a factor of 0.5 and a translation 1 unit right. Which function describes this transformation?
   F \( k(t) = 2 \ln (t + 1) \)
   G \( k(t) = 2 \ln (t - 1) \)
   H \( k(t) = 0.5 \ln (t - 1) \)
   J \( k(t) = 0.5 \ln (t + 1) \)
LESSON 7-3  Curve Fitting with Exponential and Logarithmic Models

Solve.

1. A small group of farmers joined together to grow and sell wheat in 1985. The table shows how their production of wheat increased over 20 years.

<table>
<thead>
<tr>
<th>Wheat Produced by Growers Co-op</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Years After 1985</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>Wheat (tons)</td>
<td>70</td>
<td>105</td>
<td>150</td>
<td>210</td>
<td>340</td>
<td>580</td>
</tr>
</tbody>
</table>

a. Find an exponential model for the data.

b. Use the model to predict when their wheat production will exceed 2000 tons.

2. The table shows the U.S. production of tobacco from 1997 to 2002.

<table>
<thead>
<tr>
<th>Tobacco Production</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Years After 1996</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Tobacco</td>
<td>1787</td>
<td>1480</td>
<td>1293</td>
<td>1053</td>
<td>992</td>
<td>890</td>
</tr>
<tr>
<td>(× 100,000 pounds)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Find a logarithmic model for the data.

b. Use the model to predict when tobacco production could fall below 50,000,000 pounds.

Robert recently discovered a forgotten student loan bill. The amount due after 10 years is now $10,819.33. He found some old statements and determined that after 7 years the bill was $8831.80 and after 5 years he owed $7714.03. Choose the letter for the best answer.

3. Which function models the data?
   A  $S(x) = 5000(1.07)^x$
   B  $S(x) = 1.07(5000)^x$
   C  $S(x) = 5500(1.07)^x$
   D  $S(x) = 1.07(5500)^x$

4. How much did Robert borrow initially?
   F  $5750$
   G  $5500$
   H  $5250$
   J  $5000$

5. Robert is planning to pay the loan in full next year. How much will he owe then?
   A  $12,092.14$
   B  $11,925.07$
   C  $11,869.33$
   D  $11,576.69$

6. What is the interest rate on Robert's student loan?
   F  7%
   G  6%
   H  5%
   J  4%
Problem Solving

Variation Functions

Last semester 5 of Mr. Dewayne’s students built a 7-foot sailboat in 195 working hours. The time, \( t \), that it takes for a group of students to build a sailboat varies inversely as the number of students. Mr. Dewayne uses this data to plan activities for next semester.

1. How many working hours would it take 12 students to build the same kind of sailboat?
   a. Write an equation relating the time required to the number of students working.
   b. Find the constant \( k \).
   c. Explain the meaning of \( k \) in terms of student hours.
   d. Solve the equation to answer the question.

2. How many students would be needed to build a 7-foot sailboat in 75 working hours?

Choose the letter for the best answer.

3. Which equation represents the number of hours it would take 15 students to build a 7-foot sailboat?
   A \( \frac{195}{5} = \frac{t}{15} \)
   B \( \frac{t}{195} = \frac{15}{5} \)
   C \( (195)(5) = 15t \)
   D \( 195t = (5)(15) \)

4. How many students must Mr. Dewayne get to participate in order to build a 7-foot sailboat in 65 hours?
   A 14 students
   B 15 students
   C 16 students
   D 17 students

5. Penny sells hot dogs from a cart at the beach. Her daily income, \( s \), varies directly as the number of hot dogs, \( h \), that she sells, and \( s = 255.50 \) when \( h = 73 \). Find \( h \) when \( s = 304.50 \).
   A 81
   B 83
   C 85
   D 87

6. The cost, \( c \), of hiring a contractor to build a patio varies jointly as the area, \( A \), in square feet, of the patio and the price, \( p \), per square foot of the patio tiles; and \( c = 2832 \) when \( A = 80 \text{ ft}^2 \) and \( p = 2.95 \). Find \( p \) when \( c = 4368 \) and \( A = 112 \text{ ft}^2 \).
   A \$2.95
   B \$3.25
   C \$3.45
   D \$3.60
Problem Solving

8-2 Multiplying and Dividing Rational Expressions

Anders designs a running field that consists of three concentric tracks as shown in the diagram.

1. How do the lengths of each track compare?
   a. Write an equation for the length of the inner track, \( T_1 \), in terms of radius, \( r_1 \). 
   
   b. Write an equation for the length of the middle track, \( T_2 \), in terms of radius \( r_1 \). 
   
   c. Then write a rational expression for the ratio of the length of track \( T_2 \) to the length of track \( T_1 \) in terms of radius \( r_1 \). 
   
2. Mari writes the expression \( \frac{(r_1 + 10)(r_1 - 5)}{r_1^2 - 25} \) for the ratio of the length of the outer track, \( T_3 \), to that of the middle track, \( T_2 \). Anders thinks that is the wrong expression. Simplify Mari’s expression to determine if she is correct. Explain.

3. Anders sets the radius of the inner track, \( T_1 \), at 70 meters.
   a. How many times longer is the middle track, \( T_2 \), than the inner track, \( T_1 \)?

   b. How many times longer is the outer track, \( T_3 \), than the middle track, \( T_2 \)?

   c. How many times longer is the outer track, \( T_3 \), than the inner track, \( T_1 \)?

Choose the letter for the best answer.

4. How many times as large is the area enclosed by the outer track, \( T_3 \), than the area enclosed by the inner track, \( T_1 \)?
   
   A \( \frac{10}{r_1} \) \hspace{1cm} B \( \frac{10}{r_1} \) \hspace{1cm} C \( \frac{r_1 + 10}{r_1} \) \hspace{1cm} D \( \left( \frac{r_1 + 10}{r_1} \right)^2 \)

5. What is the ratio of the area between the inner track and the outer track to the area enclosed by the inner track?
   
   A \( 20 \left( \frac{r_1 + 5}{r_1^2} \right) \) \hspace{1cm} B \( \frac{(r_1 + 10)^2 - 1}{r_1^2} \) \hspace{1cm} C \( \pi \left( \frac{r_1 + 10}{r_1} \right)^2 \) \hspace{1cm} D \( \pi \left( \frac{10}{r_1} \right)^2 \)
LESSON 8-3
Problem Solving
Adding and Subtracting Rational Expressions

Vicki and Lorena motor downstream at about 6 knots (nautical miles per hour) in their boat. The return trip is against the current, and they can motor at only about 3 knots.

1. Vicki wants to find the average speed for the entire trip.
   a. Write an expression for the time it takes to travel downstream plus the time it takes for the return trip if the distance in each direction is \( d \).

   \[
   \text{time} = \frac{d}{6} + \frac{d}{3}
   \]

   b. What is the total distance they travel downstream and upstream in terms of \( d \)?

   \[2d\]

   c. Write an expression for their average speed using the expressions for the total time and the total distance.

   \[
   \text{average speed} = \frac{2d}{\frac{d}{6} + \frac{d}{3}}
   \]

   d. Vicki says that the average speed is 4 knots. Lorena says that the average speed is 4.5 knots. Explain who is correct and why.

   Vicki is correct. Possible answer: Lorena calculated the average speed as if it took the same amount of time for each leg of the trip. Vicki took into consideration the time for each leg.

2. If they delay the return trip until the current changes direction, they can motor back at 4 knots. What is the average speed for the entire trip under these conditions?

Zak runs at an average speed of 7.0 miles per hour during the first half of a race and an average speed of 5.5 miles per hour during the second half of the race. Choose the letter for the best answer.

3. Which expression gives Zak's average speed for the entire race?
   
   A \[\frac{7 + 5.5}{2}\]
   B \[\frac{12.5(7 + 5.5)}{2}\]
   C \[\frac{38.5d}{7 + 5.5}\]
   D \[\frac{2(38.5)d}{(7 + 5.5)d}\]

4. If Zak runs the race in 1.25 hours, what is the length of the race in miles?
   
   A 3.85
   B 6.25
   C 7.7
   D 12.5

5. In a later race, Zak increased his average speed during the second half of the race to 6.0 miles per hour. What is his average speed for this race in miles per hour?
   
   A 6.42
   B 6.46
   C 6.52
   D 6.56

6. It took Zak 1.6 hours to run this later race. What is the length of this race in miles?
   
   A 5.17
   B 7.28
   C 9.55
   D 10.34
Problem Solving

Rational Functions

Members of a high school cheerleading squad plan a trip to support their robotics team at a regional competition. The trip will cost $70 per person plus a $420 deposit for the bus.

1. Find the total cost of the trip per cheerleader.
   a. Write a function that represents the total cost of the trip per cheerleader.

   \[ f(x) = \frac{70x + 420}{x} \]

   b. Graph the function on your graphing calculator. Sketch a graph to represent the function.
   c. Use your graph to determine the cost per person if 5 cheerleaders go on the trip. $154
   7 cheerleaders go on the trip. $130
   10 cheerleaders go on the trip. $112
   d. What is the horizontal asymptote of the function? What does it mean in terms of the cost per person of the trip?

2. Stanton invites the cheerleaders to support the school's dive team at their next competition. The trip will cost $145 per person plus a $1000 deposit.

   a. Write a function to represent the cost of the trip per person.

   \[ f(x) = \frac{145x + 1000}{x} \]

   b. What is the cost per person if 5 cheerleaders go on the trip? $345.00
   7 cheerleaders go on the trip? $287.90
   10 cheerleaders go on the trip? $245.00
   c. What is the increased cost for each of 10 cheerleaders to go with the dive team rather than to go with the robotics team? $133.00

Choose the letter for the best answer.

3. The deposit for the bus on the robotics trip increases to $550. By how much does the cost per person increase if 10 cheerleaders go on the trip?
   A $13  
   B $37  
   C $83  
   D $112

4. There are 15 cheerleaders signed up to go to the competition with the dive team. What is the cost per person to the nearest dollar?
   A $76  
   B $103  
   C $212  
   D $304
LESSON 8-5

Problem Solving

Solving Rational Equations and Inequalities

Norton and Jessie have a lawn service business. Sometimes they work by themselves, and sometimes they work together. They want to know if it is worthwhile to work together on some jobs.

1. Norton can mow a large lawn in about 4.0 hours. When Norton and Jesse work together, they can mow the same lawn in about 2.5 hours. Jesse wants to know how long it would take her to mow the lawn if she worked by herself.
   a. Write an expression for Jessie’s rate, using \( j \) for the number of hours she would take to mow the lawn by herself.
   b. Write an equation to show the amount of work completed when they work together.
   c. How long would it take Jessie to mow the lawn by herself?

2. Jessie can weed a garden in about 30 minutes. When Norton helps her, they can weed the same garden in about 20 minutes. Norton wants to know how long it would take him to weed the garden if he worked by himself.
   a. Write an expression for Norton’s rate, using \( n \) for the number of hours he would take to weed the garden by himself.
   b. Write an equation to show the amount of work completed when they work together.
   c. How long would it take Norton to weed the garden by himself?

Choose the letter for the best answer.

3. Norton can edge a large lawn in about 3.0 hours. Jessie can edge a similar lawn in about 2.5 hours. Which equation could be used to find the time it would take them to edge that lawn if they worked together?
   A \( \frac{1}{3} - \frac{1}{2.5} = \frac{1}{t} \)
   B \( \frac{1}{3} - \frac{1}{2.5} = t \)
   C \( \frac{1}{3} + \frac{1}{2.5} = \frac{1}{t} \)
   D \( \frac{1}{3} + \frac{1}{2.5} = t \)

4. When Jessie helps Norton trim trees, they cut Norton’s time to trim trees in half. What can be said about the time it would take Jessie to do the job alone?
   A Jessie would take the same amount of time as Norton.
   B Jessie would take half the time that Norton takes.
   C Jessie would take twice the time that Norton takes.
   D There is not enough information.
**Problem Solving**

**Radical Expressions and Rational Exponents**

Louise is building a guitar-like instrument. It has small metal bars, called frets, positioned across its neck so that it can produce notes of a specific scale on each string. The distance a fret should be placed from the bridge is related to a string’s root note length by the function \( d(n) = r \left(2^{-\frac{n}{12}}\right) \), where \( r \) is the length of the root note string and \( n \) is the number of notes higher than that string’s root note. Louise wants to know where to place frets to produce different notes on a 50-cm string.

1. Find the distance from the bridge for a fret that produces a note exactly one octave (12 notes) higher than the root note.
   a. Substitute values for \( r \) and \( n \) in the given function.
   b. How far from the bridge should the fret be placed?
   c. What fraction of the string length is the distance of this fret from the bridge?

2. Complete the table to find the distance from the bridge, for frets that produce every other note of an entire scale on this string.

<table>
<thead>
<tr>
<th>Notes Higher than the Root Note</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance of Fret from Bridge (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Choose the letter for the best answer.

3. Rafael made a ceramic cube in art class. The cube has a volume of 336 cm\(^3\). What is the side length of the cube to the nearest centimeter?
   A 7
   B 12
   C 18
   D 56

4. Yolanda has an exercise ball with a volume of 7234 in\(^3\). Find the radius of the exercise ball to the nearest inch.
   A 24
   B 21
   C 19
   D 12

5. Which formula could you use to find the area of one side of a cube if the volume were given?
   A \( A = \sqrt[3]{V} \)
   B \( A = V^{-\frac{1}{2}} \)
   C \( A = \sqrt[3]{V} \)
   D \( A = V^{-\frac{2}{3}} \)

6. A party tent in the shape of a hemisphere has a volume of 14,130 m\(^3\). What is the area of the ground that the tent covers in square meters?
   A 653.1
   B 706.5
   C 1121.5
   D 1256.0
Problem Solving

Radical Functions

On Earth the distance, \( d \), in kilometers that one can see to the horizon is a function of altitude, \( a \), in meters, and can be found using the function 
\[
d(a) = 3.56\sqrt{\frac{a}{20850}}
\]
To find the corresponding distance to the horizon on Mars, the function must be stretched horizontally by a factor of about \( \frac{9}{5} \).

1. a. Write the function that corresponds to the given transformation.
\[
d\left(\frac{9a}{5}\right)
\]

b. Use a graphing calculator to graph the function and the parent function. Sketch both curves on the coordinate plane.

c. Use your graph to determine the approximate distance to the horizon from an altitude of 100 meters:
on Earth ________________
on Mars ________________

Choose the letter for the best answer.

2. Which equation represents the radius of a sphere as a function of the volume of the sphere?
A \( r = \sqrt[3]{\frac{3\pi}{4V}} \)  
B \( r = \sqrt[3]{\frac{3V}{4\pi}} \)  
C \( r = \sqrt[3]{\frac{4V}{3\pi}} \)  
D \( r = \sqrt[3]{\frac{4\pi}{3V}} \)

3. Alice graphed a function that is found only in the first quadrant. Which function could she have used?
A \( f(x) = \sqrt{x} + 2 \)  
B \( f(x) = -\sqrt{x} \)  
C \( f(x) = \sqrt{x} + 2 \)  
D \( f(x) = \sqrt{x} - 2 \)

5. The side length of a cube can be represented by \( s = \sqrt[3]{\frac{T}{6}} \), where \( T \) is the surface area of the cube. What transformation is shown by \( s = \sqrt[3]{\frac{T}{3}} \)?
A Horizontal compression by a factor of 0.5
B Horizontal stretch by a factor of 3
C Vertical compression by a factor of 2
D Vertical stretch by a factor of 0.5

4. Harry made a symmetrical design by graphing four functions, one in each quadrant. The graph of which function is in the third quadrant?
A \( f(x) = 4\sqrt{x} \)  
B \( f(x) = 4\sqrt{-x} \)  
C \( f(x) = -4\sqrt{x} \)  
D \( f(x) = -4\sqrt{-x} \)

6. The hypotenuse of a right isosceles triangle can be written \( H = \sqrt{2x^2} \), where \( x \) is the length of one of the legs. Which function models the hypotenuse when the legs are lengthened by a factor of 2?
A \( H = \sqrt{2x^2} + 2 \)  
B \( H = \sqrt{2x^2} + 4 \)  
C \( H = \sqrt{4x^2} \)  
D \( H = \sqrt{8x^2} \)
LESSON
6-8
Problem Solving
Solving Radical Equations and Inequalities

The formula \( s = \sqrt{\frac{30f}{d}} \) can be used to estimate the speed, \( s \), in miles per hour that a car is traveling when it goes into a skid, where \( f \) is the coefficient of friction and \( d \) is the length of the skid marks in feet.

1. How does the speed vary as the length of the skid marks?

2. Kody skids to a stop on a street with a speed limit of 35 mi/h. His skid marks measure 52 ft, and the coefficient of friction is 0.7. Kody says that he was driving only about 30 mi/h. Kody wants to prove that he was not speeding.
   a. Solve the equation for \( d \) in terms of \( s \).
   b. How long would the skid marks be if he had been driving at a speed of 35 mi/h?
   c. Was Kody speeding or not? Explain how you know.
   d. Find his actual speed.

3. Ashley skids to a stop on a street with a speed limit of 15 mi/h to avoid a dog who runs into the street about 20 ft ahead of her. Ashley claims to have been going less than 15 mi/h. The coefficient of friction is 0.7.
   a. If Ashley were driving the speed limit, by what distance would she have missed the dog?
   b. If Ashley were driving less than 10 mi/h, by what distance would she have missed the dog?

Choose the letter for the best answer.

4. Barney was driving at 25 mi/h. A car pulls out 30 ft ahead of him. Which statement is true?
   A Barney hits the car.
   B Barney stops less than a foot from the car.
   C Barney misses the car by 3 ft.
   D Barney’s skid marks measure 23 ft.

5. On a busy highway with a speed limit of 70 mi/h, a truck ahead of Verna jackknifes across the road. Verna skids to a stop 10 ft short of the truck. Her skid marks measure 260 ft. Was Verna speeding?
   A Yes; her speed was 73.9 mi/h.
   B Yes; her speed was 75.3 mi/h.
   C No; her speed was 70 mi/h.
   D No; her speed was only 63 mi/h.
Problem Solving
Multiple Representations of Functions

Yvonne opened a new video game store. The table shows a record of her sales for the first five weeks. To break even, she needs to sell at least $12,500 worth of merchandise each week. She assumes the sales trend will continue and wants to know what to expect over the next weeks.

1. Graph the data using weeks as the independent variable and sales as the dependent variable.

2. Yvonne thinks she can model her sales data using a quadratic function. Is she correct? How do you know?

3. Use a graphing calculator to perform the appropriate regression on the data. Write the equation that models the data.

4. What sales can Yvonne expect in week 6?

5. When will her sales exceed $11,000 per week?

6. When will she break even?

7. When will sales be twice the sales of week 1?

Choose the letter for the best answer.

8. Which equation represents a steady increase of $420 per week in sales from week 5 on?
   A. \( y = -420x + 10,210 \)
   B. \( y = 420x + 10,210 \)
   C. \( y = -5x^2 + 420x + 10,210 \)
   D. \( y = -420x^2 + 10,210x \)

9. During which week will Yvonne break even if the sales pattern changes and sales in week 6 and week 7 are $10,640 and $11,080, respectively?
   F. Week 9
   G. Week 10
   H. Week 11
   J. Week 12

Weekly Store Sales

<table>
<thead>
<tr>
<th>Week</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8470</td>
</tr>
<tr>
<td>2</td>
<td>8920</td>
</tr>
<tr>
<td>3</td>
<td>9360</td>
</tr>
<tr>
<td>4</td>
<td>9790</td>
</tr>
<tr>
<td>5</td>
<td>10,210</td>
</tr>
</tbody>
</table>
Problem Solving

Lesson 9-2

Piecewise Functions

Roscoe earns $9.50 per hour at the woodcrafts store for up to 40 hours per week. For each hour over 40 hours he earns $13.00 per hour. Company policy limits his hours to no more than 60 per week. Roscoe wants to know how much he can earn in a week.

1. Complete the table to show his earnings for 30, 40, 50, and 60 hours per week.

<table>
<thead>
<tr>
<th>Hours per Week (t)</th>
<th>Roscoe’s Earnings, E(t)</th>
<th>Total Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours ≤ 40</td>
<td>($9.50 per h)</td>
<td></td>
</tr>
<tr>
<td>Hours &gt; 40</td>
<td>($13.00 per h)</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>$285</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>$285</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Graph earnings as a function of hours worked, using the data from the table. Roscoe thinks the points lie in a straight line. Is he correct?

3. Draw line segments to join the points, including the point that represents earnings for 0 hours worked. Describe the graph in terms of line segments and the slope of each segment. Explain the meaning of the slope in terms of rate of pay.

4. Use the slope and a point on each line segment to write a piecewise function for earnings $E(t)$ as a function of hours worked $t$.

Choose the letter for the best answer.

5. How much will Roscoe earn if he works 56.5 hours in one week?
   A. $594.50
   B. $610.00
   C. $625.50
   D. $734.50

6. Roscoe earned $471 last week. How many hours did he work?
   F. 45
   G. 47
   H. 49
   J. 51
Problem Solving

Transforming Functions

In March, Mei researches the sports clubs in her area. One club charges different rates according to the number of hours of use per month. The rate scale is modeled by the piecewise function below, where \( x \) is the number of hours of use.

\[
f(x) = \begin{cases} 
5x + 105 & \text{if } 0 \leq x < 12 \\
165 & \text{if } 12 \leq x \leq 20 \\
7(x - 20) + 165 & \text{if } x > 20 
\end{cases}
\]

Mei learns that all rates will increase by 10% in June. In September the club is planning to add a $5 monthly energy fee.

1. Describe the transformation of the fees in March, \( f(x) \), to the fees in June, \( g(x) \).

2. Write the rules for the rates that will be effective in June.

3. Describe the transformation of the fees in June, \( g(x) \), to the fees in September, \( h(x) \).

4. Write the rules for the rates that will be effective in September.

Mei considers several options. Choose the letter for the best answer.

5. How much will she pay if she joins and uses the club for 10 hours in March?
   A $95.00
   B $110.00
   C $155.00
   D $165.00

6. How much will she pay if she joins and uses the club for 10 hours in July?
   F $115.50
   G $165.50
   H $170.50
   J $175.50

7. How much will she pay if she joins and uses the club between 12 and 20 hours in August?
   A $215.50
   B $181.50
   C $165.00
   D $155.00

8. How much will she pay if she joins and uses the club for 25 hours in October?
   F $192.50
   G $208.00
   H $215.50
   J $225.00
Andy is buying a new laptop computer. The store is offering a 15% discount on the model he wants. Andy also has a certificate from the manufacturer good for $120 off his next computer purchase. Andy wants to know whether he will pay less if the certificate amount is deducted before or after the discount is applied.

1. Write a function, $D$, to represent the price of the computer after the discount is applied. Use $p$ to represent the original price.

2. Write a function, $R$, to represent the price after the certificate amount is deducted.

3. Find the composite function $R(D(p))$ and then describe what it means in terms of the final price of the computer.

4. Find the composite function $D(R(p))$ and then describe what it means in terms of the final price of the computer.

5. Will Andy pay less if the certificate amount is deducted before or after the discount is applied? Explain.

Emily has a coupon from her favorite store for $5 off any item this month. A sales tax of 8% is applied after the value of a coupon or discount is deducted. Let $p$ represent the original price, $t(p)$ the price after tax, and $d(p)$ the price after the value of a coupon or discount is deducted. Choose the letter for the best answer.

6. Which function describes the final price of an item with price, $p$?
   A $td(p)$  
   B $dt(p)$  
   C $d(t(p))$  
   D $t(d(p))$

7. Emily picks out a backpack with a price tag of $p$. Which expression gives the amount that Emily will pay?
   F $1.08(p - 5)$  
   G $(p - 5) + 0.08$  
   H $0.92(p - 5)$  
   J $1.08p - 5$

8. If Emily uses her coupon to buy a pair of jeans that has been marked down to $50, what will she pay?
   A $41.40$  
   B $45.00$  
   C $48.60$  
   D $49.00$

9. A month later Emily buys a jacket on sale at 30% off. Which expression gives the amount that she pays after tax?
   F $(p - 0.3) + 1.08$  
   G $1.08(0.7p)$  
   H $0.7p + 1.08p$  
   J $0.92(p - 0.3)$
A juice drink manufacturer is designing an advertisement for a national sports event on its cans. The lateral surface area of the cans is given by the function \( L(h) = 2.5\pi h \), where \( h \) is the height of the can. The total surface area of the can is given by the function \( T(h) = 2.5\pi(h + 1.25) \).

1. The graphic designer needs to know how the height of the can varies as a function of the lateral surface area.

   a. Find the inverse, \( h(L) \), of the function \( L(h) \).

   b. Explain the meaning of the inverse function.

   c. If the lateral surface area of one can is 35.34 \( \text{in}^2 \), what is the height of this can?

Choose the letter for the best answer.

2. The manufacturer produces cans in different sizes. The height of one can is 5.5 in. The designer is planning to use only half the lateral surface area of this can. What is this area?
   - A 8.8 \( \text{in}^2 \)
   - B 11.0 \( \text{in}^2 \)
   - C 21.6 \( \text{in}^2 \)
   - D 43.2 \( \text{in}^2 \)

3. The designer is studying the possibility of using the total surface area of each can. Which function gives height, \( h(T) \), as a function of total surface area, \( T \)?
   - F \( h(T) = \frac{T}{2.5\pi} - 1.25 \)
   - G \( h(T) = \frac{T}{2.5\pi} + 1.25 \)
   - H \( h(T) = (2.5\pi)(T - 1.25) \)
   - J \( h(T) = (2.5\pi)(T + 1.25) \)

4. The total surface area of one size of can is 45.16 \( \text{in}^2 \). What is the height of this can?
   - A 3.5 in.
   - B 4.5 in.
   - C 5.5 in.
   - D 6.5 in.

5. The designer updates an old advertisement that covers the lateral surface area, \( L \), of a can to create a new advertisement that covers the total surface area, \( T \), of the can. Which function gives this area?
   - F \( T = (2.5\pi)(L + 1.25) \)
   - G \( T = (2.5\pi)(L - 1.25) \)
   - H \( T = L - 2.5\pi(1.25) \)
   - J \( T = L + 2.5\pi(1.25) \)
Problem Solving

Modeling Real-World Data

The table shows the population of Lincoln Valley over the last 7 years. The town council is developing long range plans and is considering how the population might grow in the future if the current trend continues.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>1049</td>
<td>1137</td>
<td>1229</td>
<td>1326</td>
<td>1434</td>
<td>1542</td>
<td>1662</td>
</tr>
</tbody>
</table>

1. What is the independent variable? What is the dependent variable? Assign \( x \) or \( y \) to each variable.

2. Make a scatter plot of the data. Do the data form a linear pattern? For this to be true, explain what must be true about finite differences.

3. Use the table of data.
   a. Find the first differences.
   b. Find the second differences.
   c. Find the third differences.
   d. Find the ratios between \( y \)-values.

4. What kind of function will best describe the data? Justify your conclusion.

Choose the letter for the best answer.

5. Which function best models the given data?
   A \( y = 101.9x + 932.1 \)
   B \( y = 3.1x^2 + 77.0x + 969.6 \)
   C \( y = 996.6x^{0.233} \)
   D \( y = 974.9(1.08)^x \)

6. Predict the population of Lincoln Valley in 2012.
   F 2270
   G 2450
   H 2650
   J 2860
Problem Solving

Introduction to Conic Sections

Hunter and Max draw a grid over a map of their town so that they can determine precisely where to set up a broadcast station. Hunter decides that locations A and B should be the endpoints of the diameter of a circle around the broadcast station.

1. Locate the center of the circle using the Midpoint Formula and points A and B.

2. Find the radius and the circumference of the circle.
   a. Name the coordinates of two points you can use with the Distance Formula to find the radius.
   b. Use the Distance Formula to find the radius.
   c. What is the circumference of the circle?

3. Hunter says that point C is within the circle. Max says that point C is on the circumference of the circle.
   a. Explain how they can use the Distance Formula to determine the answer.
   b. Who is correct? Why?

A broadcast station is set up at the location with coordinates (0, 2). The transmissions just reach a location with coordinates (8, 8). Choose the letter for the best answer.

4. Which point is farthest from the point (8, 8) and still within range of the broadcast station?
   A (-8, -4)    C (0, 10)
   B (-8, -8)    D (10, 2)

5. What is the maximum distance the broadcast transmissions will reach?
   F 6 units
   G 8 units
   H 9 units
   J 10 units

6. What is the area the broadcast transmissions will cover?
   A $20\pi$
   B $64\pi$
   C $100\pi$
   D $196\pi$

7. The broadcast transmissions just reach the point (x, 0). What is the value of x?
   F 8.4
   G 8.8
   H 9.2
   J 9.8
LESSON 10-2  Problem Solving

Circles

When Claire started her consulting business, she decided not to accept any clients located more than 10 miles from her home. On the graph below, her home is located at \( (5, 1) \), and her prospective clients are represented by the letters \( P \) through \( W \).

1. Claire needs to know which prospective clients are located within a 10-mile radius of her home.
   a. Write the equation of a circle with center \( (5, 1) \) and radius 10.
   
   ______________________________

   b. Which prospective clients are located within 10 miles of Claire’s home?
   
   ______________________________

   c. Which prospective clients are located more than 10 miles from Claire’s home?
   
   ______________________________

2. Claire finds that she isn’t earning enough to pay the expenses from her consulting business and she needs to find some additional clients. She decides that she will drive up to 15 miles from her home to visit a client. Which prospective clients are within a 15-mile radius of her home?
   a. Write the equation of a circle with center \( (5, 1) \) and radius 15.
   
   ______________________________

   b. Which prospective clients are located within 15 miles of Claire’s home?
   
   ______________________________

   c. How far away is a prospective client located at \( (10, 15) \)?
   
   ______________________________

Claire rents an office at location \( (-2, -3) \). Choose the letter for the best answer.

3. Which of the prospective clients \( P \) through \( W \) are located more than 15 miles from Claire’s office?
   A  \( R, V \)  
   B  \( R, S, V \)  
   C  \( Q, R, V \)  
   D  \( Q, R, S, V \)  

4. Which location is within 5 miles of Claire’s office?
   F  Post office at \( (2, 3) \)  
   G  Library at \( (2, -3) \)  
   H  Marina at \( (-3, 2) \)  
   J  Swimming pool at \( (-2, 3) \)
Problem Solving

Ellipses

Lenore and Zane study a drawing of an ornamental bridge such as the one shown at right. It shows an elliptical arch that spans a narrow strait of water. The arch they are studying can be modeled by the following equation.

\[ \frac{x^2}{182.25} + \frac{y^2}{132.25} = 1 \]

1. Lenore wants to know the dimensions of the arch.
   a. Which term in the equation defines the horizontal axis?
   b. Write a comparative statement to show whether the major axis of the arch is horizontal or vertical.
   c. Lenore says that the width of the bridge is 13.5 feet. Is she correct? Explain.
   d. Write an expression for the height of the bridge.

2. Zane notes that this bridge is hardly high enough to pass under while standing up in a moderate-size boat. If he were to build a bridge, it would be at least 1.3 times as wide and twice as high.
   a. Find the vertices and co-vertices of Zane's bridge design.
   b. Write an equation for the design of Zane's bridge using his minimum dimensions.

Lenore finds architectural drawings for other bridges. Choose the letter for the best answer.

3. The equation for the arch of a bridge at the entrance to a wildlife park is \( \frac{x^2}{225} + \frac{y^2}{324} = 1 \). What is the width of this bridge?
   A 36 ft
   B 30 ft
   C 18 ft
   D 15 ft

4. The width and the height of one arch leading into part of an old town are both 17 feet. What is the equation for this bridge?
   F \( \frac{x^2}{72.25} + \frac{y^2}{289} = 1 \)
   G \( \frac{x^2}{72.25} + \frac{y^2}{72.25} = 1 \)
   H \( \frac{x^2}{289} + \frac{y^2}{72.25} = 1 \)
   J \( \frac{x^2}{289} + \frac{y^2}{289} = 1 \)
Problem Solving

Hyperbolas

A brochure for a new amusement park describes the design of the
towers that characterize the park. The outline of the central and
largest tower can be modeled by the hyperbola \( \frac{x^2}{625} - \frac{y^2}{2025} = 1 \), with
dimensions in feet. The smaller towers are scaled-down versions of
the central tower, and so their dimensions are in proportion to those
of the central tower.

1. What is the diameter of the central tower at its narrowest part?
   a. Explain how to determine whether the transverse axis of a hyperbola
      is horizontal or vertical.
   b. Find the vertices and co-vertices.
   c. How can you use these data to answer the question?
   d. What is the diameter of the tower at its narrowest part?

2. The vertices of a smaller tower are \((10, 0)\) and \((-10, 0)\).
   a. Name the coordinates of the co-vertices.
   b. What is the equation for the outline of the
      smaller tower?

3. How do the asymptotes of the hyperbolas of the larger central tower
   and the smaller tower compare?
   a. Find the asymptotes of the two towers.
   b. Compare the asymptotes of the two towers.

Choose the letter for the best answer.

4. The hyperbola that models a third
tower has vertices \((0, 9)\) and \((0, -9)\)
and focus \((0, 15)\). The solution to
which of the following equations gives
the denominator of the \(y^2\) term in the
equation of the hyperbola?
   A \( 15^2 = a^2 - 9^2 \)
   B \( 15^2 = a^2 + 18^2 \)
   C \( 15^2 = b^2 + 9^2 \)
   D \( 15^2 = b^2 - 18^2 \)

5. The outline of a tower at the park’s
   Welcome Center can be modeled by the
equation \( \frac{x^2}{25} - \frac{y^2}{31} = 1 \). What are the
   asymptotes of the hyperbola?
   F \( y = \pm 0.31x \)
   G \( y = \pm 0.56x \)
   H \( y = \pm 1.8x \)
   J \( y = \pm 3.24x \)
Problem Solving

Rodrigo and Juan are constructing a model of the mirror grid in the solar wind concentrator used during the Genesis space mission to collect charged solar particles. They know that a cross section of the mirror grid can be modeled by the equation \( y = \frac{x^2}{80} \), with dimensions in centimeters.

1. Draw a graph of the cross-sectional shape of the mirror grid.
   a. What are the coordinates of the vertex of the parabola?
   b. Find the distance, \( p \), from the vertex to both the focus and the directrix of the parabola. Write an equation for \( p \).
   c. Write an equation to represent the axis of symmetry.
   d. Find the coordinates of the focus.
   e. Write an equation to represent the directrix.
   f. Sketch a graph of the cross section of the mirror grid including the focus and the directrix.

2. Juan wants to construct a different parabolic mirror grid. He changes the focus to \( (0, 5) \) and the directrix to \( y = -5 \).
   a. Explain why you can use the Distance Formula to find the equation for this parabola.
   b. Find the equation for the new parabola.

Choose the letter for the best answer.

3. Juan writes an equation for a parabola with vertex \( (0, 0) \) and directrix \( x = -4 \). Which equation could he have written?
   A \( x = \frac{y^2}{16} \)  
   B \( x = \frac{y^2}{4} \)
   C \( y = \frac{x^2}{16} \)  
   D \( y = \frac{x^2}{4} \)

4. If the equation of the mirror grid in the Genesis solar wind concentrator is \( y - 2 = \frac{1}{15} (x - 3)^2 \), where is the vertex of the parabola?
   F \( (2, 3) \)  
   G \( (2, -3) \)  
   H \( (-3, 2) \)  
   J \( (3, 2) \)
Problem Solving
Identifying Conic Sections

At a bungee-jumping contest, Gavin makes a jump that can be modeled by the equation \( x^2 - 12x - 12y + 84 = 0 \), with dimensions in feet.

1. Gavin wants to know how close he came to the ground during his jump.
   a. Classify the shape of his path. Identify the values for the coefficients of each term, and determine what conic section models his path.

   b. Write the equation of his path in standard form by completing the square.

   c. Which point on the path identifies the lowest point that Gavin reached? What are the coordinates of this point? How close to the ground was he?

2. Nicole makes a similar jump that can be modeled by the equation \( x^2 - 4x - 8y + 84 = 0 \). She wants to know whether she got closer to the ground than Gavin and by how much.
   a. Write the equation of Nicole's path in standard form.

   b. How close to the ground did she get?

   c. Did Nicole get closer to the ground than Gavin?

The design for a new auto racetrack can be modeled by the equation \( x^2 + 4y^2 - 20x - 32y + 160 = 0 \), with dimensions in kilometers.
Tracey tests the track. Choose the letter for the best answer.

3. What is the standard form of the equation for the path of the racetrack?
   A \( \frac{(x - 10)^2}{1^2} + \frac{(y - 4)^2}{2^2} = 1 \)
   B \( \frac{(x - 10)^2}{2^2} + \frac{(y - 4)^2}{1^2} = 1 \)
   C \( \frac{(x - 4)^2}{2^2} + \frac{(y - 10)^2}{1^2} = 1 \)
   D \( \frac{(x - 4)^2}{1^2} + \frac{(y - 10)^2}{2^2} = 1 \)

4. While driving around the track, what is the greatest distance that Tracey will reach from the center of the track?
   F 1 km
   G 2 km
   H 10 km
   J 16 km
LESSON 10-7
Problem Solving
Solving Nonlinear Systems

Jim sets a course in his fishing boat that can be modeled by the equation \(4x^2 + 9y^2 = 36\). Janice has her sailboat on a path that can be modeled by the equation \(x - 2 = \left(\frac{1}{8}\right)y^2\).

1. Janice wonders whether there is a danger of them colliding.
   a. Explain which conic section models each boat's path and why.

   b. What is the maximum number of points of intersection between two such paths?

   c. To solve this system of equations using the quadratic formula, what expression can be substituted for \(y^2\) in the equation of the path of Jim's boat?

   d. Rewrite the equation of the path of Jim's boat in terms of \(x\).

   e. Solve for \(x\) using the quadratic formula.

   f. Is there a possibility of collision? Is so, name the coordinates of the point(s) of intersection.

2. Janice changes course so that her sailboat now follows a path modeled by the equation \(y^2 - x^2 = 1\) in quadrants I and II. Is there now a possibility of collision?
   a. Sketch the system of equations represented by Jim's boat's course and Janice's boat's new course.
   b. Use your graph to determine if there is now a possibility of collision. If so, give the coordinates of the point(s).

A trawler is on a course modeled by the equation \(2x + y + 3 = 0\). Choose the letter for the best answer.

3. Which solution set represents possible points of collision between Jim's boat and the trawler's course?
   A \((-2.2, 1.37)\)
   B \((-0.5, -2)\)
   C \((-2.2, 1.37); (-0.5, -2)\)
   D There is no possible collision point.

4. Janice sees the trawler. Which equation models a path that avoids a possible collision of her boat with the trawler?
   F \(x - 1 = 0.5y^2\)
   G \(y = 0.5x + 2\)
   H \(y^2 - x^2 = 1\)
   J \(4x^2 + 3y^2 = 12\)
Rosalie is looking at locks. The label combination lock confuses her. She wonders about the number of possible permutations or combinations a lock can have.

1. She looks at one circular lock with 12 positions. To open it she turns the dial clockwise to a first position, then counterclockwise to a second position, then clockwise to a third position
   a. Write an expression for the number of 3-position codes that are possible, if no position is repeated.
   b. Explain how this represents a combination or a permutation.

2. Rosalie looks at cable locks. Each position can be set from 0 to 9. How many different codes are possible for each lock if no digits are repeated in each code?
   a. a 3-digit cable lock
   b. a 4-digit cable lock
   c. a 6-digit cable lock

3. Rosalie needs 2 cable locks, but there are 13 types of locks to choose from.
   a. In how many ways can she choose 2 different locks?
   b. Explain how this represents a permutation or a combination.

4. Explain why you think Rosalie might be confused by the label combination lock.

Rosalie wants to lock her bicycle near the library. There are 7 slots still open in the bike rack. Choose the letter for the best answer.

5. Rosalie arrives at the same time as 2 other cyclists. In how many ways can they arrange their bikes in the open slots?
   A 7  B 35  C 210  D 343

6. Suppose Rosalie arrived just ahead of the 2 other cyclists and selected a slot. In how many ways can the others arrange their bikes in the open slots?
   F 2  G 15  H 24  J 30
As part of a grant to improve bus routes to and from school, Hogan and Jane gather traffic flow statistics for one intersection. They make a table to show their findings for between 7:45 A.M. and 8:00 A.M. on a Monday morning.

1. Analyze the statistics.
   a. Write and evaluate an expression for $P(N)$, the probability that a vehicle will turn north.
   b. Write and evaluate an expression for the probability that a vehicle will turn north or go straight through the intersection.
   c. Write and evaluate an expression for the probability that a vehicle will not turn north.

2. The police department gathers statistics on Tuesday. Officers count a total of 608 vehicles, of which 380 go straight through the intersection, 76 turn north, and the rest turn south.
   a. What is the probability that a vehicle will turn north?
   b. What is the probability that a vehicle will turn north or go straight through the intersection?
   c. What is the probability that a vehicle will not turn north?

3. Does this represent theoretical or experimental probability? Explain.

### Traffic Direction

<table>
<thead>
<tr>
<th>Traffic Direction</th>
<th>Number of Vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight through</td>
<td>282</td>
</tr>
<tr>
<td>Turn north</td>
<td>94</td>
</tr>
<tr>
<td>Turn south</td>
<td>188</td>
</tr>
</tbody>
</table>

### Math Assessment Survey

<table>
<thead>
<tr>
<th>Activity</th>
<th>Group projects</th>
<th>Keep a Journal</th>
<th>Multiple Choice</th>
<th>Word Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Response</td>
<td>57</td>
<td>18</td>
<td>35</td>
<td>10</td>
</tr>
</tbody>
</table>

A teacher surveys students on how they would prefer to have work assessed in math class. Choose the letter for the best answer.

4. What is the probability that a randomly chosen student prefers assessment through a group project?
   - A $\frac{1}{12}$
   - B $rac{53}{120}$
   - C $\frac{19}{40}$
   - D $\frac{21}{40}$

5. Which expression gives the probability that a randomly chosen student will not want multiple-choice questions?
   - F $\frac{7}{24}$
   - H $1 - \frac{7}{24}$
   - G $1 + \frac{7}{24}$
   - J $\frac{35}{24} - \frac{7}{24}$
Problem Solving

Independent and Dependent Events

The table shows student participation in different sports at a high school. Suppose a student is selected at random.

<table>
<thead>
<tr>
<th>Sports Participation by Grade</th>
<th>Track</th>
<th>Volleyball</th>
<th>Basketball</th>
<th>Tennis</th>
<th>No Sport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 9</td>
<td>12</td>
<td>18</td>
<td>15</td>
<td>9</td>
<td>66</td>
</tr>
<tr>
<td>Grade 10</td>
<td>6</td>
<td>20</td>
<td>12</td>
<td>2</td>
<td>95</td>
</tr>
<tr>
<td>Grade 11</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>5</td>
<td>61</td>
</tr>
<tr>
<td>Grade 12</td>
<td>7</td>
<td>6</td>
<td>10</td>
<td>12</td>
<td>50</td>
</tr>
</tbody>
</table>

1. What is the probability that a student is in grade 10 and runs track?
   a. Find the probability that a student is in grade 10, \( P(10) \).
   b. Find the probability that a student runs track, given that the student is in grade 10, \( P(Tr \mid 10) \).
   c. Find \( P(10 \text{ and } Tr) = P(10) \cdot P(Tr \mid 10) \).

2. What is the probability that a student is in grade 12 and runs track or plays tennis?
   a. Find the probability that a student is in grade 12, \( P(12) \).
   b. Find the probability that a student runs track or plays tennis, given that the student is in grade 12, \( P(Tr \text{ or } Te \mid 12) \).
   c. Find \( P(12 \text{ or } (Tr \text{ or } Te)) \).

3. During a fire drill, the students are waiting in the parking lot. What is the probability that one student is in grade 12 and runs track or plays tennis, and the student standing next to her is in grade 10 and runs track?
   a. Find the probability for the first student.
   b. Find the probability for the second student.
   c. Find the probability for the event occurring.
   d. Are these events independent or dependent? Explain.

Samantha is 1 of 17 students in a class of 85 who have decided to pursue a business degree. Each week, a student in the class is randomly selected to tutor younger students. Choose the letter for the best answer.

4. What is the probability of drawing a business student one week, replacing the name, and drawing the same name the next week?
   A 3.4  
   B 0.2  
   C 0.04  
   D 0.002

5. What is the probability of drawing Samantha’s name one week, not replacing her name, and drawing the name of another business student the next week?
   \( \frac{1}{85} \cdot \frac{16}{84} \)  
   \( \frac{17}{85} \cdot \frac{16}{84} \)  
   \( \frac{1}{85} \cdot \frac{17}{84} \)  
   \( \frac{17}{85} \cdot \frac{17}{84} \)
LESSON 11-4
Problem Solving
Compound Events

Of 100 students surveyed, 44 are male and 54 are in favor of a change to a 9-period, 4-day school week. Of those in favor, 20 are female. One student is picked at random from those surveyed.

1. What is the probability that the student is male or favors the change? Use the Venn diagram.
   a. What is represented by the total of \( A + B \)?
   b. What is represented by the total of \( B + C \)?
   c. How many of those in favor of the change are male?
   d. Find the values for \( A \), \( B \), and \( C \) and label the diagram.
   e. Write and evaluate an expression for the probability that the student is male or favors the change.

2. What is the probability that the student is female or opposes the change?
   a. How many students are female?
   b. How many students oppose the change?
   c. If you draw a Venn diagram to show females and those opposed to the change, what is the meaning and value of the overlapping area?
   d. Write and evaluate an expression for the probability that the student is female or opposes the change.

3. Of the students surveyed, 27 plan to start their own businesses. Of those, 18 are in favor of the change to the school week. Write and evaluate an expression for the probability that a student selected at random plans to start his or her own business or favors the change.

Sean asks each student to cast a vote for the type of class he or she would prefer. Of the students, 55% voted for online classes, 30% voted for projects, and 15% voted for following the textbook. Choose the letter for the best answer.

4. Which description best describes Sean’s experiment?
   A Simple events
   B Compound events
   C Mutually exclusive events
   D Inclusive events

5. What is the probability that a randomly selected student voted for online classes or projects?
   F \( \frac{33}{200} \)
   H \( \frac{1}{4} \)
   G \( \frac{7}{10} \)
   J \( \frac{17}{20} \)
LESSON 11-5

Problem Solving
Measures of Central Tendency and Variation

Each week, Damien records the miles per gallon for his car, to the nearest whole number. Over a period of 10 weeks, the data are 18, 17, 19, 18, 18, 25, 29, 30, 26, 19. He wants to arrange and summarize his data so that he can analyze it.

1. Make a box-and-whisker plot of his data.
   a. Order the data from least to greatest.
   b. Identify the minimum, maximum, median, first quartile, and third quartile.
   c. Use the number line to make a box-and-whisker plot of the data.
      Find and label the interquartile range.

   d. Explain what the interquartile range represents in terms of the car’s miles per gallon.

2. Find the standard deviation for the data.
   a. Write an equation and solve to find the mean.
   b. Complete the table to show the difference between the mean and each data value, and the square of that difference.

<table>
<thead>
<tr>
<th>Data Value, x</th>
<th>18</th>
<th>17</th>
<th>19</th>
<th>18</th>
<th>18</th>
<th>25</th>
<th>29</th>
<th>30</th>
<th>26</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>x - \bar{x}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x - \bar{x})^2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. Explain how to use the data from the table to find the standard deviation.

   d. What is the standard deviation for the data?

   e. Explain what the standard deviation represents in terms of the car’s miles per gallon.

3. Damien thinks that the standard deviation is a more reliable measure of dispersion than the interquartile range. Is he correct? Explain.
LESSON 11-6
Problem Solving
Binomial Distributions

Sales records for the snack machines show that 1 out of every 6 students buys a bag of trail mix. There are 5 students waiting to use the machines. Melanie uses the formula for binomial probability, $P(r) = nC_r p^r q^{n-r}$, to determine the number of students expected to buy trail mix.

1. What is the probability of exactly 3 students buying a bag of trail mix?
   a. What is the probability of each student buying a bag of trail mix?
   b. Define each variable used in the formula and give its value.
   c. Write the binomial formula using these values.
   d. Solve the equation to give the probability of exactly 3 students buying a bag of trail mix.

2. What is the probability of at least 1 student buying a bag of trail mix?
   a. Describe a method to solve involving the sum of probabilities.
   b. Describe a method to solve that uses the formula $P(E) + P(not\ E) = 1$.
   c. Use either method to determine the probability of at least 1 student buying a bag of trail mix.

3. After school, 4 students line up to buy snacks from the machine. What is the probability that they will all buy a bag of trail mix?

Sports drinks are purchased by 3 out of 4 students using the snack machines. There are 3 students at the machines now. Choose the letter for the best answer.

4. Which expression gives the probability of exactly 2 students buying an energy drink?
   A $P(2) = 4C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^1$
   B $P(2) = 4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^1$
   C $P(3) = 4C_3 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^2$
   D $P(3) = 4C_3 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^2$

5. What is the probability that at least 2 of the students will buy an energy drink?
   F 42%
   G 50%
   H 75%
   J 84%
Tina is working on some home improvement projects involving repeated tasks. She wants to analyze her work patterns.

1. Tina is hammering nails into wallboard. With the first hit, a nail goes in 13.5 millimeters; with the second, it goes in an additional 9 mm; with the third, it goes in an additional 6 mm; and with the fourth it goes in 4 mm further. Suppose this pattern continues. Predict how far the nail would go in with the seventh hit.

   a. Complete the table to find first differences, second differences, and ratios.

<table>
<thead>
<tr>
<th>Distance</th>
<th>13.5 mm</th>
<th>9 mm</th>
<th>6 mm</th>
<th>4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratios</td>
<td>2/3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Differences</td>
<td>-4.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second Differences</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. How do you know whether the rule for the sequence of distances that the nail goes in is linear, quadratic, or exponential?

   c. Write a possible rule for \(a_n\), the \(n^{th}\) term in the sequence.

   d. If this pattern continues, how far would the nail go in with the seventh hit?

2. Tina builds a fence for her neighbor. It takes her 10 minutes to pound the first fence post into the ground. The neighbor predicts that Tina should improve her time on each successive fence post according to the rule \(a_n = F - 2(n - 1)\), where \(F\) is the time for the first fence post, and \(a_n\) is the time it takes to pound in the \(n^{th}\) post.

   a. Use the rule to find the time it should take Tina to pound each of the first 4 fence posts into the ground.

   b. If the rule that describes Tina’s time on each successive post is \(a_n = F - 1.5^{n-1}\), how long will it take her to pound the fourth fence post into the ground?

The label on Pete’s blue jeans states that, when washed, the jeans will lose 5% of their color. Choose the letter for the best answer.

3. Which rule describes the percent of color left in the blue jeans after \(n\) washings?

   A \(a_n = 100(0.05)^n\)

   B \(a_n = 100(0.95)^n\)

   C \(a_n = 100(0.95)n\)

   D \(a_n = 100(0.05)n\)

4. How much of the original color will be left after 8 washings?

   F 66%

   G 60%

   H 40%

   J 34%
Problem Solving

Series and Summation Notation

Todd joins a fitness club. During the first week of training, his biceps increase by 4 millimeters. The trainer says Todd can expect his biceps to continue to increase each week, but only by about 90% of the increase of the week before.

1. Todd wants to know how much his biceps will increase in 8 weeks.
   a. Write a rule for the \( k \)th term in the sequence that represents the amount of muscle increase each week.
   b. Write the summation notation using \( \sum \) for the first 8 terms.
   c. Expand the series and evaluate to find the amount by which Todd’s biceps will increase in 8 weeks.

2. Todd thinks that, if he works out extra hard each week, his biceps should increase by at least an additional half-millimeter each week. If this is true, how much will his biceps increase after 8 weeks?
   a. Write a rule for the \( k \)th term in the sequence that represents the minimum amount of muscle increase each week.
   b. Use summation notation to represent the minimum total amount of muscle increase after 8 weeks.
   c. What is the minimum increase in size in Todd’s biceps after 8 weeks?
   d. At this rate, how many weeks would it take to reach or exceed the total muscle increase predicted by the trainer?

Rodrigo puts his change into a bowl each evening. On Monday he puts 2 quarters in the bowl and decides to try and increase the amount each evening by at least 10 cents over the evening before.

Choose the letter for the best answer.

3. Which series represents the minimum amount in the bowl Saturday morning?
   A \[ \sum_{k=1}^{5} 0.1(0.5)^{k-1} \]
   B \[ \sum_{k=1}^{5} 0.5(0.1)^{k-1} \]
   C \[ \sum_{k=1}^{5} 0.5 + 0.1(k-1) \]
   D \[ \sum_{k=1}^{5} 0.1 + 0.5(k-1) \]

4. What is the minimum amount in the bowl the following Monday morning?
   F $3.50
   G $5.60
   H $6.80
   J $7.60
Problem Solving

Arithmetic Sequences and Series

Violet has the book and the assignment for her literature class. Counting today, Monday, as day 1, she sees that she must read through page 385 by day 12, and through page 665 by day 20.

1. If Violet reads an equal number of pages each day, how many pages will she have read by this Friday?
   a. First write a rule for the \( n \)th term of an arithmetic sequence that represents the number of pages, \( a_n \), that she will have read after \( n \) days.
   b. Explain how to find the common difference by using the rule with the given data.
   c. What is the common difference, and what does it mean in terms of Violet’s assignment?
   d. How many pages must Violet read today, Monday? Explain how you know this.
   e. Use the rule to find the number of pages Violet will have read by Friday.

2. Violet looks at the table of contents in her book. She sees that each of the first 6 chapters is 2 pages longer than the preceding chapter, with the first chapter having 10 pages.
   a. How many pages are in the sixth chapter? Write a rule for the number of pages, \( a_n \), in chapter \( n \). Then solve for \( n = 6 \).
   b. How many pages are in the first 6 chapters? Use the formula for the sum of the first \( n \) terms of an arithmetic sequence.

Choose the letter for the best answer.

3. Violet reads the first 385 pages by day 6. Which expression gives the minimum pages she must read each day to finish 665 pages by day 20?
   A \[ 6 \left( \frac{665 + 385}{2} \right) \]
   B \[ \frac{665 - 385}{6} \]
   C \[ \frac{14}{2} \left( \frac{665 + 385}{2} \right) \]
   D \[ \frac{665 - 385}{14} \]

4. Page 385 of Violet’s book is 15 pages before the end of a 40-page chapter. If chapter 1 contains 10 pages, which chapter contains page 385?
   F Chapter 15
   G Chapter 16
   H Chapter 19
   J Chapter 25
Crystal works at a tree nursery during the summer. She wonders why the lower branches of one particular type of tree drop off. The nurseryman explains that each layer of branches absorbs about 10% of the sunlight and lets the rest through to the next layer. If a layer receives less than 25% of the sunlight, those branches will drop off.

1. Crystal counts 7 distinct healthy layers on one tree. She wants to know how many more layers the tree will grow before starting to lose layers of branches.
   a. Write the rule for the \( n \)th term, \( a_n \), of a geometric sequence.
   
   b. If \( a_n \) represents the percent of sunlight reaching layer \( n \), what is the value of \( a_1 \)? How do you know?
   
   c. What is the value of \( r \)? What does it represent?
   
   d. Write the rule to find the percent of sunlight reaching layer 7. Solve for \( a_7 \).
   
   e. About how many layers of branches will this tree have before a bottom layer drops off?

2. The nurseryman points to a denser type of tree and states that only about 75% of sunlight gets through to each lower layer, but that a layer of this type of tree needs only about 15% of the original sunlight to survive.
   a. What percent of sunlight gets through to layer 7 of this tree?
   
   b. How many layers of branches could this type of tree support?

Jackson usually runs 8 laps around the football field and consistently completes the first lap in 3 minutes. During one practice session, his coach notes that it takes him 15% longer to complete each lap than the previous lap. Choose the letter for the best answer.

3. How long does it take Jackson to complete the eighth lap?
   A 5.25 min
   B 6.03 min
   C 6.93 min
   D 7.98 min

4. How long does it take Jackson to complete all 8 laps?
   F 9.18 min
   G 20.39 min
   H 41.18 min
   J 61.18 min
Diego drops a ball from different heights. He determines that after each bounce the ball rises to 80% of its previous height. He wants to find out how the total vertical distance that the ball falls before it comes to rest varies with the height from which it is dropped.

1. Diego drops the ball from a height of 8 feet and calculates the total vertical distance that the ball falls.
   a. Find the total vertical distance that the ball has fallen by the third bounce, $a_1 + a_2 + a_3$.
   b. Is this a converging or diverging series? How do you know?
   c. Write a rule and solve for the total vertical distance that the ball falls.

2. Next Diego drops the ball from a height of 6 feet. What is the total vertical distance that the ball falls from this initial height?

3. Complete the table.

<table>
<thead>
<tr>
<th>Drop Height (ft)</th>
<th>Total Vertical Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

4. Write an equation to show the relationship between the drop height and the total vertical distance that the ball travels.

Kali notices that each year after the first year, a particular plant in her backyard adds only about 25% of the previous year’s new growth to its height. The first year’s growth from a seed was 3 feet. Choose the letter for the best answer.

5. Which statement describes the plant’s ultimate height?
   A Its height will converge to 4 ft.
   B Its height will converge to 4.5 ft.
   C Its height will converge to 12 ft.
   D Its height will not converge.

6. In which year will new growth be less than 0.1 ft?
   F First year
   G Second year
   H Third year
   J Fourth year
Problem Solving

LESSON 13-1
Right-Angle Trigonometry

Kayla is fishing near the ferry landing near her home. She wonders how far the ferries actually travel when they cross the river. To find out, she puts a fishing pole upright on the riverbank directly across from the ferry landing. Then she walks down the bank 180 yards and measures an angle of 75° between the lines to her fishing pole and the ferry landing on the opposite bank.

1. Find the distance directly across the river.
   a. On the diagram, name the side of the triangle that represents the distance Kayla wants to determine.
   b. Write a trigonometric function that relates the known distance and angle to the required distance.
   c. Find the distance to the nearest yard directly across the river.

2. Kayla walks another 200 yards down the riverbank. At this point the ferries seem to come straight at her at an angle of 60° to the line to her fishing pole. If the boats travel along this line to compensate for the current, how far do they travel?
   a. How far is she from her fishing pole now?
   b. Write the trigonometric function that relates this distance and angle to the required distance, $c$.
   c. What is the distance that the ferries travel if they travel along this line?

At a hot-air balloon festival, Luis watches a hot-air balloon rise from a distance of 200 yards. Choose the letter for the best answer.

3. From Luis’s position, the balloon seems to hover at an angle of elevation of 50°. Which trigonometric function gives the height of the balloon, $h$?
   A $200 \sin 50°$
   B $200 \tan 50°$
   C $\frac{200}{\cos 50°}$
   D $\frac{200}{\cot 50°}$

4. After a short while, the balloon seems to hover at an angle of 75°. How high is it off the ground now?
   F 193 yd
   G 207 yd
   H 746 yd
   J 773 yd

Olivia has a pool slide that makes an angle of 25° with the water. The top of the slide stands 4.5 feet above the surface of the water. Choose the letter for the best answer.

5. How far out into the pool will the slide reach?
   A 2.1 ft
   B 5.0 ft
   C 7.6 ft
   D 9.7 ft

6. The slide makes a straight line into the water. How long is the slide?
   F 5.0 ft
   G 7.6 ft
   H 9.7 ft
   J 10.6 ft
LESSON 13.2
Problem Solving
Angles of Rotation

Isabelle and Karl agreed to meet at the rotating restaurant at the top of a tower in the town center. The restaurant makes one full rotation each hour.

1. Isabelle waits for Karl, who arrives 30 minutes later. Through how many degrees does the restaurant rotate between the time that Isabelle arrives and the time that Karl arrives?

2. Since the restaurant is busy, they don’t get menus for another ten minutes. How much farther has the restaurant rotated?

3. By the time they are served dinner, the restaurant has rotated to an angle 30° short of its orientation when Isabelle arrived.
   a. Write an expression for the length of time that Isabelle has been there.
   b. How long has she been there?
   c. How long has Karl been there?
   d. How far has the restaurant rotated since Karl arrived?

4. When their bill comes, it includes a note that the restaurant has rotated 840° since Isabelle arrived.
   a. How long has it been since they were served dinner?
   b. How many rotations has the restaurant made since Isabelle arrived?
   c. How far is the restaurant from its orientation when Karl arrived?

5. On their way out, they stop and look at a map. A museum is located at a point northeast of the tower with coordinates (3, 2).
   a. Write the trigonometric function for the angle that the line from the tower to the museum makes with a line due north from the tower.
   b. Isabelle says that the museum is exactly 4 kilometers from the tower. How much farther north is the museum than the tower?

An advertisement on a kiosk near the bus stop rotates through 765° while Aaron waits for his bus. Choose the letter for the best answer.

6. What is the difference in the orientation of the advertisement between when Aaron arrived at the bus stop and when the bus came?
   A 45°
   B 90°
   C 315°
   D 405°

7. If the advertisement rotates at a rate of one rotation every 10 minutes, how long does Aaron wait for his bus?
   F Less than 2 min
   G Between 2 min and 10 min
   H Between 10 min and 20 min
   J More than 20 min
Problem Solving

The Unit Circle

Gabe is spending two weeks on an archaeological dig. He finds a fragment of a circular plate that his leader thinks may be valuable. The arc length of the fragment is about $\frac{1}{6}$ the circumference of the original complete plate and measures 1.65 inches.

1. A similar plate found earlier has a diameter of 3.14 inches. Could Gabe’s fragment match this plate?
   a. Write an expression for the radius, $r$, of the earlier plate.
   b. What is the measure, in radians, of a central angle, $\theta$, that intercepts an arc that is $\frac{1}{6}$ the length of the circumference of a circle?
   c. Write an expression for the arc length, $S$, intercepted by this central angle.
   d. How long would the arc length of a fragment be if it were $\frac{1}{6}$ the circumference of the plate?
   e. Could Gabe’s plate be a matching plate? Explain.

2. Toby finds another fragment of arc length 2.48 inches. What fraction of the outer edge of Gabe’s plate would it be if this fragment were part of Gabe’s plate?

The diameter of a merry-go-round at the playground is 12 feet. Elijah stands on the edge and his sister pushes him around. Choose the letter for the best answer.

3. How far does Elijah travel if he moves through an angle of $\frac{5\pi}{4}$ radians?
   A 12.0 ft   C 23.6 ft
   B 15.1 ft   D 47.1 ft

4. Through what angle does Elijah move if he travels a distance of 80 feet around the circumference?
   F $\frac{40}{3}$ radians   H $\frac{40}{3}$ radians
   G $\frac{80}{3}$ radians   J $\frac{20}{3}$ radians

Virgil sets his boat on a 1000-yard course keeping a constant distance from a rocky outcrop. Choose the letter for the best answer.

5. If Virgil keeps a distance of 200 yards, through what angle does he travel?
   A $5\pi$ radians   C 10 radians
   B 5 radians   D $10\pi$ radians

6. If Virgil keeps a distance of 500 yards, what fraction of the circumference of a circle does he cover?
   F $\frac{1}{\pi}$   H $\frac{3}{4\pi}$
   G $\frac{\pi}{3}$   J $\frac{3\pi}{4}$
Problem Solving

Inverses of Trigonometric Functions

Rafe is concerned that some recently constructed buildings in his town do not comply with code restrictions. New buildings are limited to a maximum of 40 feet in height.

1. When working with angles of elevation from his eye level, Rafe realizes that he must allow for his own height, 5 feet 9 inches, in his calculations. Explain how he can do this.

2. On the building plans, the height of the new bank is 33 feet. Rafe calculates what the angle of elevation should be from 100 feet away if the bank is 33 feet tall.
   a. Label the diagram to show the height of the bank that Rafe will use in his calculations. Mark the angle of elevation.
   b. Write a trigonometric function for the assumed angle of elevation, \( \theta \).
   c. To the nearest tenth of a degree, what is the assumed angle of elevation?
   d. Using a clinometer, Rafe measures the angle of elevation to be 14.2°. Does the bank comply with the building code? Explain.

3. On the plans, the height of the new inn is 39 feet. Rafe finds the angle of elevation and compares it to what he measures.
   a. Predict the angle of elevation of the highest point on this building from a distance of 100 feet and allowing for Rafe's height.
   b. Predict the angle of elevation of the highest point on a building that is 40 feet tall, allowing for Rafe's height.
   c. If Rafe measures an angle of elevation of 19.0°, how does the height of the inn compare to its declared height of 39 feet? Explain.

4. Carrie says the angle of elevation of the top of the flagpole at school is 55.9° from a distance of 20 feet away and allowing for her height of 5 feet 6 inches.
   a. Write and evaluate an expression for the height of the flagpole to the nearest tenth of a foot.
   b. What should be the angle of elevation if Rafe measures it from a distance of 50 feet away? Write and evaluate an expression.
Problem Solving
The Law of Sines

In the middle of town, State and Elm streets meet at an angle of 40°. A triangular pocket park between the streets stretches 100 yards along State Street and 53.2 yards along Elm Street. Hoa and Cat walk around the pocket park every day at lunchtime.

1. Hoa would like to know the area of the pocket park.
   a. Write a formula for the area of the pocket park using the given dimensions.
   b. What is the area of the pocket park to the nearest tenth of a square yard?

2. Cat determines that the total distance around the pocket park is 221.6 yards.
   a. Write and evaluate an expression for the length of the park along West Avenue.
   b. Use the Law of Sines to find ∠S to the nearest degree, the angle that West Avenue makes with State Street.
   c. Write an expression for the angle that West Avenue makes with Elm Street.

3. West Avenue makes angles of 55° with Main Street and 80° with Third Street. The distance from Main to Third along West Avenue is 40 yards. Hoa contracts to design a pocket park for the acute-angled triangular area enclosed by these streets.
   a. What is the measure of the third angle of the triangular area?
   b. Write and evaluate an expression for the distance from West Avenue to Third Street along Main Street.

Choose the letter for the best answer.

4. Hoa wants to plant palm trees 8 feet apart along the side of the park on Third Street. Which expression gives the number of trees she will need?
   A \( \frac{40 \sin 55°}{\sin 45°} \)
   B \( \frac{5 \sin 55°}{\sin 45°} \)
   C \( \frac{5 \sin 45°}{\sin 55°} \)
   D \( \frac{40 \sin 45°}{\sin 55°} \)

5. What is the area of the new pocket park that Hoa is designing?
   F 913 yd²
   G 1004 yd²
   H 1207 yd²
   J 1398 yd²
LESSON
Problem Solving

13-6
The Law of Cosines

Standing on a small bluff overlooking a local pond, Clay wants to calculate the width of the pond.

1. From point C, Clay walks the distances $\text{CA}$ and $\text{CB}$. Then he measures the angle between these line segments.
   a. Use the Law of Cosines to write an equation for the distance $\text{AB}$.
      
   b. What is the distance to the nearest meter from $A$ to $B$?

2. From another point $F$, Clay measures 20 meters to $D$ and 50 meters to $E$. Reece says that last summer this area dried out so much that he could walk the 49 meters from $D$ to $E$.
   a. Use the Law of Cosines to write an equation for the measure of the angle between $\text{DF}$ and $\text{EF}$.
      
   b. What is the measure of this angle?

3. Reece tells Clay that when the area defined by $\triangle DEF$ dries out, it becomes covered with a grass native to this area of the country. Clay wants to know the area of this section.
   a. Use Heron’s Formula to write an expression for the area.
      
   b. Find the area to the nearest tenth of a square meter.

A local naturalist says that a triangular area on the north sides of the pond is a turtle habitat. The lengths of the sides of this area are 15 meters, 25 meters, and 36 meters. Choose the letter for the best answer.

4. What is the area of this triangular turtle habitat?
   A 150.7 $\text{m}^2$
   B 213.2 $\text{m}^2$
   C 1012.9 $\text{m}^2$
   D 3075 $\text{m}^2$

5. Which expression gives the measure of the angle between the sides of the habitat that measure 15 meters and 25 meters?
   F $\cos^{-1} (-0.595)$
   G $\cos (-0.595)$
   H $\cos (0.595)$
   J $\cos^{-1} (0.595)$
Problem Solving
Graphs of Sine and Cosine

As a result of the tide, the depth of the water in the bay varies with time. According to the harbormaster, the depth can be modeled by the function \( d(t) = 2.5 \cos \left( \frac{\pi}{6} (t - 2) \right) + 4 \), where \( t \) is the number of hours since midnight, and \( d \) is the depth in meters.

1. Identify the following features of the graph of the function.
   - Amplitude _______________________
   - Period _______________________
   - Phase shift _______________________
   - Vertical shift _______________________
   - What is the maximum depth and when does that occur?
     _______________________
   - What is the minimum depth and when does that occur?
     _______________________

2. Graph the function for 24 hours.

3. How many high tides occur in 24 hours?

4. Jim thinks that the amplitude and the vertical shift should be reversed.
   - Write the equation for this new function.
   - Explain how water depth and frequency of tides would change using Jim’s function.

5. Petra thinks that the original function is correct, except for the period, which should be 24. If so, how many hours after midnight would the first high and low tides occur?

Jim looks at equations for the depth of water in different locations.
Choose the letter for the best answer.

6. Which function represents a location that will dry out?
   - A \( d(t) = 5 \cos \frac{\pi}{12} (t - 1) + 4 \)
   - B \( d(t) = 4 \cos \frac{\pi}{6} (t - 2) + 4.5 \)
   - C \( d(t) = 3 \cos \frac{\pi}{6} (t - 3) + 5 \)
   - D \( d(t) = 2 \cos \frac{\pi}{4} (t - 4) + 5.5 \)

7. How deep is the water at 1:00 A.M. in a location where the water depth \( t \) hours after midnight is modeled by \( d(t) = 1.25 \cos \frac{\pi}{12} (t - 1) + 6 \)?
   - F 4.75 m
   - G 5.75 m
   - H 7.25 m
   - J 8.25 m
Problem Solving

Graphs of Other Trigonometric Functions

As the sign outside the community youth center rotates, the light on it traces a path on the center’s wall. The distance in feet of the light to the point on the wall where it shines varies with time, \( t \), and can be modeled by the function \( g(t) = 9 \csc \left( \frac{3\pi}{4} t \right) \), where \( t \) is measured in seconds.

1. Identify the period of the function.
   a. What is the coefficient of \( t \) in the equation of the function?
   b. Write and evaluate an expression for the period of the function.
   c. Explain what the period of the function means in terms of the time for one rotation of the light.
   d. How does the graph of the function vary based on the period?

2. a. This function has asymptotes at points where what other function is equal to 0?
   b. What are the asymptotes of the function?

3. What is the significance of 9 in the equation for the function?

4. Graph the function using all the information about the function.

Toni changes the rotation rate of the light so that it rotates once every 4 seconds. Choose the letter for the best answer.

5. Which equation represents the distance of the light to the point on the wall where it shines, as a function of time, \( t \)?
   A \( g(t) = 9 \csc \left( \frac{\pi}{4} t \right) \)
   B \( g(t) = 9 \csc \left( \frac{\pi}{2} t \right) \)
   C \( g(t) = 9 \csc \left( \frac{\pi}{2} t \right) \)
   D \( g(t) = 9 \csc \left( 4\pi t \right) \)

6. Which pair of parameters describes this new function?
   F period: 4 s; vertical asymptotes at \( \pm 2\pi \) s
   G period: 2 s; vertical asymptotes at \( \pm \pi \) s
   H period: 2 s; horizontal asymptotes at \( \pm 2\pi \) s
   J period: 4 s; horizontal asymptotes at \( \pm \pi \) s
The advertisement for a new shoe promises runners less slip. The coefficient of friction ($\mu$) between concrete and a new material used in the sole of this shoe is 1.5. The force of friction that causes slip is equal to $mg \sin \theta$, where $m$ is a runner’s mass and $g$ is the acceleration due to gravity. The force that prevents slip is $\mu mg \cos \theta$.

1. Mukisa wants to know what this new shoe will do for his performance.
   a. At the instant of slip, the force that causes slip is equal to the force that prevents it. Write an equation to show this relationship.
   _______________________________________________________________________
   b. Rewrite the equation with trigonometric expressions on one side of the equation, and substitute the coefficient of friction for the new material.
   _______________________________________________________________________
   c. Use a trigonometric identity to rewrite the equation using only the tangent function.
   _______________________________________________________________________
   d. Solve for $\theta$, the angle at which the new shoe will start to slip.
   _______________________________________________________________________

2. Mukisa wonders how his old shoes compare to these new shoes. The label on the box of his old shoes states that the coefficient of friction is 1.3.
   a. Write an equation to find the angle at which a shoe with a coefficient of friction of 1.3 will slip.
   _______________________________________________________________________
   b. At what angle will this shoe start to slip?
   _______________________________________________________________________
   c. Suggest how the new shoes might improve Mukisa's performance.
   _______________________________________________________________________

An advertisement states, “Use this wax and your skis will slide better than silk on silk!” The coefficient of friction for silk on silk is 0.25. The coefficient of friction for waxed wood on wet snow is 0.14. The equation $mg \sin \theta = \mu mg \cos \theta$ can be used to find the angle at which a material begins to slide. Choose the letter for the best answer.

3. Which expression gives the angle at which silk begins to slide on silk?
   A $\cos^{-1}(0.25)$
   B $\sin\left(\frac{1}{0.25}\right)$
   C $\tan\left(\frac{1}{0.25}\right)$
   D $\tan^{-1}(0.25)$

4. At what angle would a waxed wooden ski begin to slide on wet snow?
   F $2^\circ$
   G $8^\circ$
   H $12^\circ$
   J $14^\circ$
Caitlin is designing a rotating graphic for her website. She explores what it will look like if it flashes in different positions.

1. Use rotation matrices to find the coordinates of the figure \( \text{PQRS} \) after a 60° rotation about the origin.
   a. Write matrices for a 60° rotation and for the points \( P, Q, R, \) and \( S \) in the figure.
      \[
      R_{60°} = \begin{bmatrix}
      \cos 60° & -\sin 60° \\
      \sin 60° & \cos 60° 
      \end{bmatrix}
      \]
      \[
      PQR = \begin{bmatrix}
      P_x & P_y \\
      Q_x & Q_y \\
      R_x & R_y \\
      S_x & S_y 
      \end{bmatrix}
      \]
   b. Find the matrix product. \( R_{60°} \times PQR = \)
   c. Name the coordinates of \( P', Q', R', S' \) to the nearest hundredth after a 60° rotation.

2. a. Write the matrix product that gives the coordinates of the figure after a rotation of 150°.
      \[
      R_{150°} \times PQR = \]
   b. Name the coordinates of \( P''Q''R''S'' \) to the nearest hundredth.

3. Graph the two rotations on the plane above to see where Caitlin's rotating graphic will flash. Label \( R' \) and \( R'' \).

Choose the letter for the best answer.

4. Which expression gives the exact value of \( \sin 150° \)?
   A \( \sin 90° \cos 60° + \cos 90° \sin 60° \)
   B \( \sin 90° \cos 60° - \cos 90° \sin 60° \)
   C \( \cos 180° \cos 30° + \sin 180° \sin 30° \)
   D \( \cos 180° \cos 30° - \sin 180° \sin 30° \)

5. What is the exact value of \( \cos 75° \)?
   F \( \frac{\sqrt{2} - \sqrt{6}}{2} \)
   G \( \frac{\sqrt{6} + \sqrt{2}}{4} \)
   H \( \frac{\sqrt{6} - \sqrt{2}}{4} \)
   J \( \frac{\sqrt{6} + \sqrt{2}}{2} \)
Problem Solving

14-5

Double-Angle and Half-Angle Identities

Chris set a new personal best in football with a 35-yard kick. He wants to improve his distance. His coach suggests using the formula
\[ d(\theta) = \frac{3v_0^2 \sin \theta \cos \theta}{16}, \]
which gives the horizontal distance, \( d \), in yards, as a function of the angle of elevation, \( \theta \), at which the ball is kicked with an initial velocity of \( v_0 \) yards per second.

1. What is the angle of elevation of the ball when Chris kicks it with an initial velocity of 20 yards per second and the ball covers 35 yards.
   a. What double-angle identity can you use to rewrite the function in terms of \( \sin^2 \theta \)?
   b. Rewrite the function in terms of \( \sin^2 \theta \).
   c. Substitute the known values in the equation.
   d. Solve for \( \sin^2 \theta \).
   e. What is the angle at which he kicked the ball?

2. Chris wonders how the horizontal distance would change if he kicked the ball with the same velocity but at a different angle.
   a. Complete the table to show the horizontal distance, to the nearest hundredth yard, for the given angles.
   b. At what angle must Chris kick the ball to cover the greatest horizontal distance?

<table>
<thead>
<tr>
<th>Angle of Elevation (°)</th>
<th>Horizontal Distance (yd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td></td>
</tr>
</tbody>
</table>

3. What is the least angle at which Chris must kick the ball if he wants to cover at least 30 yards horizontally?

4. Chris thinks he can kick the ball with an initial velocity of 18 yards per second and make a horizontal distance of 40 yards. Is he correct? Explain.

Choose the letter for the best answer.

5. If Chris kicks the ball with an initial velocity of only 15 yards per second and an angle of elevation of 45°, what is the maximum horizontal distance the ball will cover?
   A 12.06 yd  
   B 14.92 yd  
   C 21.09 yd  
   D 29.83 yd

6. At what angle can Chris kick the ball to cover a horizontal distance of 36 yards, if he kicks the ball with an initial velocity of 20 yards per second?
   F 18.5°  
   G 36.9°  
   H 44.1°  
   J 73.7°
Problem Solving

LESSON 14-6
Solving Trigonometric Equations

Jon watches his sister Jessica skateboard. When she jumps and launches herself into the air with an initial speed of $v_0$ feet per second, her path in terms of time, $t$, in seconds, is represented by these equations.

Equation 1 \[ x(t) = v_0 t \cos \theta \]

Equation 2 \[ y(t) = v_0 t \sin \theta - 16t^2 \]

The first equation models the horizontal distance, $x(t)$, that the skateboarder travels, and the second equation models the vertical height, $y(t)$, that the skateboarder attains.

1. Jessica attains a height of 4.7 feet above the launch and landing ramps after 1 second. Her initial velocity is 25 feet per second. Find the angle of her launch.

   a. Which equation can you use with the given information to solve for $\theta$?

   b. Substitute the known values and solve for $\theta$.

   c. What is Jessica’s height above the launch and landing ramps after 0.5 second?

   d. What distance has Jessica traveled after 1 second?

2. Jon attains a height of 5.2 feet above the launch and landing ramps after 1 second. His initial velocity is 28 feet per second. Find the angle of his launch.

   a. Write and evaluate an expression for Jon’s launch angle.

   b. Write and evaluate an expression for his height above the launch and landing ramps after 0.5 second.

Choose the letter for the best answer.

3. How far does Jon travel in 1 second?
   \[ \begin{array}{cccc}
   \text{A} & 14.5 \text{ ft} & \text{C} & 18.3 \text{ ft} \\
   \text{B} & 16.7 \text{ ft} & \text{D} & 21.2 \text{ ft} \\
   \end{array} \]

4. Jessica increases her initial velocity to 30 feet per second. She attains a height of 5.5 feet after 1 second. Which expression represents her launch angle?
   \[ \begin{array}{cccc}
   \text{F} & \cos^{-1} \left( \frac{10.5}{30} \right) & \text{H} & \sin^{-1} \left( \frac{10.5}{30} \right) \\
   \text{G} & \cos^{-1} \left( \frac{21.5}{30} \right) & \text{J} & \sin^{-1} \left( \frac{21.5}{30} \right) \\
   \end{array} \]