Apply the Perpendicular Bisector Theorem and Its Converse

Find each measure.

A. \(DF\)

Since \(CD = CE\), and \(\ell \perp \overline{DE}\), \(\overline{CF}\) is the ________ ________ of \(DE\) by the Converse of the ________ ________ Theorem.
Therefore, \(DF = FE\) because of the definition of a ________ ________.

Substitute 3 for \(FE\).

\(DF = _____\)

B. \(VU\)

\(TU = VU\) because of the ________ ________ Theorem. Substitute the given measures for \(TU\) and \(VU\) and solve for \(x\).

\[
12x + 3 = ________
\]

\[
12x - 12x + ____ + 5 = 14x - 5 + 5 - 12x
\]

\[
8 = ____x
\]

\[
____ = x
\]

Substitute the value of \(x\) to find \(VU\).

\[
14x - 5
\]

\[
= 14(____) - 5
\]

\[
= _____
\]

Apply the Angle Bisector Theorem

Find \(QR\).

\(QR = RS\) because of the ________ ________ Theorem.
Substitute 46 for \(RS\).

\(QR = ________\)
Ready to Go On? Skills Intervention

5A 5-2 Bisectors of Triangles

Find these vocabulary words in Lesson 5-2 and the Multilingual Glossary.

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>concurrent</td>
<td>point of concurrency</td>
<td>circumcenter of a triangle</td>
</tr>
<tr>
<td>circumscribed</td>
<td>incenter of a triangle</td>
<td>inscribed</td>
</tr>
</tbody>
</table>

Using Properties of Perpendicular Bisectors

PC, PD, and PB are the perpendicular bisectors of \( \triangle LMN \).

A. Find PM.

\( P \) is the ________ of \( \triangle LMN \).

By the ___________________ Theorem, \( P \) is _________ from the vertices of \( \triangle LMN \), so \( PL = PM \).

Substitute 27.7 for PL. \( PM = \) ______

B. Find DN.

By the definition of a _______ ________, \( DM = DN \).

Substitute 24.8 for DM. \( DN = \) ______

Finding the Circumcenter of a Triangle

Find the circumcenter of \( \triangle BCD \) with vertices \( B(8, 0), C(0, -6), \) and \( D(0, 0) \).

Graph the triangle.

The equation for the perpendicular bisector of \( DB \) is \( x = \) ___.

The equation for the perpendicular bisector of \( DC \) is \( y = \) ___.

Find the intersection of the two equations. This point is the circumcenter of \( \triangle BCD \): (___, -3).

Using Properties of Angle Bisectors

WT and WS are angle bisectors of \( \triangle STU \). Find \( \text{m} \angle TSV \).

Since \( WS \) is the bisector of \( \angle TSV \), \( \text{m} \angle ____ = 2(\text{m} \angle TSW) \).

Substitute 18° for \( \text{m} \angle TSW \).

\( \text{m} \angle TSV = 2(____)° = ____° \)
Using the Centroid to Find Segment Lengths

In $\triangle QRS$, $RP = 12$ and $SB = 16$. Find each length.

A. $SP$

$P$ is called the ______ of the triangle because it is the point of intersection of the ______ of the triangle. The Centroid Theorem states that the centroid of a triangle is located ______ of the distance from each vertex to the ______ of the opposite side.

$SP = \frac{2}{3}(____)$ Centroid Theorem

$SP = \frac{2}{3}(16)$ Substitute the known value for $SB$.

$SP = ______$ Simplify.

B. $RA$

$RP = \frac{2}{3}(____)$ Centroid Theorem

$____ = \frac{2}{3}(____)$ Substitute known values for $RP$.

To solve the equation, multiply both sides by ______.

$____ = RA$ Solve for $RA$.

C. $AP$

$PA = RA - ______$

$PA = ______ - ______ = ______$ Substitute known values and solve.
A median of a triangle is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.

Sandra cuts a triangle with vertices \( W(0, 4), X(5, 0), \) and \( Y(10, 2) \) from grid paper. At what coordinates should she place the tip of a pencil to balance the triangle?

Understand the Problem
1. The \[\text{________} \] of a triangle is called the \textit{center of gravity} because it is the point where a triangular region will \[\text{________}. \]
2. The answer to this problem will be the coordinates of the \[\text{________}. \]

Make a Plan
3. First, graph the triangle.
4. The centroid of the triangle is the point of intersection of the \[\text{________} \] of the triangle.
5. Find the equations for two of the \[\text{________} \] and find the point of \[\text{________}. \]

Solve
6. Use the Midpoint Formula, \( M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \), to find the midpoint, \( M \), of \( \overline{WY} \).
   \[ M = \left( \frac{0 + \underline{\text{________}}}{2}, \frac{4 + \underline{\text{________}}}{2} \right) = \left( \underline{\text{________}}, \underline{\text{________}} \right) \]
7. Use the Midpoint Formula to find the midpoint, \( N \), of \( \overline{WX} \).
   \[ N = \left( \frac{0 + \underline{\text{________}}}{2}, \frac{4 + \underline{\text{________}}}{2} \right) = \left( \underline{\text{________}}, \underline{\text{________}} \right) \]
8. \( \overline{XM} \) is a \[\text{________} \] line. Its equation is \( x = \underline{\text{________}} \).
9. \( \overline{YN} \) is a \[\text{________} \] line. Its equation is \( y = \underline{\text{________}} \).
10. The coordinates of the centroid are \[\text{________} \].

Look Back
11. Find the midpoint of \( \overline{XY} \). \[\text{________} \]
12. Draw the line containing \( W \) and the midpoint. Does the point \( (5, 2) \) lie on the line? \[\text{________} \]

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Using the Triangle Midsegment Theorem
Find each measure.

A. Find \( BC \).

\( BC \), is a \underline{___________} of \( \triangle MNO \) because it is a segment that joins the \underline{___________} of two sides of the triangle.

According to the Triangle Midsegment Theorem, \( BC = \frac{1}{2}(\text{____}) \).

Since \( AM = 15 \), \( OA = \text{____} \), and \( OM = \text{____} \).

Substitute 30 for \( OM \). \( \text{____} = \frac{1}{2}(\text{____}) \)

Simplify to find \( BC \). \( BC = \text{____} \)

B. Find \( m\angle CBA \).

According to the Triangle Midsegment Theorem, \( \overline{BC} \parallel \text{____} \).

What kind of angles are \( \angle OAB \) and \( \angle CBA \)? \underline{__________________________}

By the Alternate Interior Angles Theorem, you know that \( m\angle OAB = m\angle \text{____} \).

Substitute 74° for \( m\angle OAB \) to find \( m\angle CBA \).

\( m\angle CBA = \text{____} \)

C. Find \( JL \).

\( \overline{XY} \) is a \underline{___________} of \( \triangle JKL \) because it is a segment that joins the \underline{___________} of two sides of the triangle.

According to the Triangle Midsegment Theorem, \( XY = \frac{1}{2}(\text{____}) \).

\( \text{____} = \frac{1}{2}JL \) Substitute 52 for \( XY \).

\( \text{____}(52) = \text{____}(\frac{1}{2}JL) \) Multiply both sides of the equation by 2 to find \( JL \).

\( JL = \text{____} \) Simplify to find \( JL \).
The Triangle Midsegment Theorem can be used to determine measures that are difficult to determine.

Karol wants to find the distance across the pond. He records the measurements in the diagram shown. Find the distance across the pond.

**Understand the Problem**

1. You can use triangle ___________ to make indirect measurements of distances.
2. A midsegment of a triangle is a ___________ that joins the ___________ of two sides of the triangle.
3. The Triangle Midsegment Theorem states that a midsegment of a triangle is ___________ to a side of the triangle, and that its length is _______ the length of that side.

**Make a Plan**

4. Use the given information to determine whether $\overline{MN}$ is a midsegment of $\triangle______$.
5. If $\overline{MN}$ is a midsegment, apply the ________________ Theorem to find _____, the distance across the pond.

**Solve**

6. Since $JM = MK = 105$, $M$ is the ___________ of ____.
7. Since $JN = NL = 158$, $N$ is the ___________ of ____.
8. $\overline{MN}$ is the ___________ of $\triangle JKL$, and by the Triangle Midsegment Theorem, $MN = \frac{1}{2}(____)$.
9. Substitute 91 for $MN$. $$\frac{1}{2}KL = __$$
10. Simplify to find $KL$. $KL = ____$
11. What is the distance across the pond? _________

**Look Back**

12. Work backward to check your answer. Divide $KL$ by 2. $$\frac{KL}{2} = ____$$ Does this value equal $MN$? _____
Ready to Go On? Quiz

5-1 Perpendicular and Angle Bisectors
Find each measure.

1. \(DC\)  
   \[
   \begin{array}{c}
   A \quad B \\
   C \quad D
   \end{array}
   \]

2. \(BC\)  
   \[
   \begin{array}{c}
   A \quad B \\
   C \quad D
   \end{array}
   \]

3. \(LM\)  
   \[
   \begin{array}{c}
   J \quad K \\
   L \quad M
   \end{array}
   \]

4. Write an equation in point-slope form for the perpendicular bisector of the segments with endpoints \(X(3, -1)\) and \(Y(5, 7)\).

5-2 Bisectors of Triangles

5. \(PC\) and \(PB\) are angle bisectors of \(\triangle ABC\). Find the following:
   a. \(m\angle BCP\)  
   b. the distance from \(P\) to \(\overline{BC}\)

6. Find the circumcenter of \(\triangle MNO\) with vertices \(M(0, -6), N(-4, 0)\) and \(O(0, 0)\).

5-3 Medians and Altitudes of Triangles

7. In \(\triangle XYZ\), \(ZL = 63, NP = 14\). Find the following:
   a. \(ZP\)  
   b. \(PY\)  
   c. \(NY\)
8. Melissa cuts a triangle with vertices at coordinates (0, 6), (4, 0), and (8, 10). At what point should she place the tip of a pencil to balance the triangle?

9. Find the orthocenter of \( \triangle JKL \) with vertices \( J(2, 1) \), \( K(9, 1) \), and \( L(4, 6) \).

5-4 The Triangle Midsegment Theorem

10. Find the following:
   a. \( EF \)
   b. \( AB \)
   c. \( m\angle EDF \)

11. What is the distance \( RQ \) across the pond?
**Ready to Go On? Enrichment**

**Special Segments in Triangles**

Answer each question.

1. The circumcenter, the orthocenter, and the centroid of a triangle are collinear. The line on which they lie is called the Euler Line of the triangle. \( \triangle PQR \) has vertices \( P(-3, 3) \), \( Q(5, 7) \), and \( R(1, -5) \). Find the equation of the Euler Line.

2. Find the value of \( x \) and find the length of each segment in the figure at right.

\( \triangle ABC \) at right is an isosceles triangle. \( M \) is the midpoint of \( AC \) and \( N \) is the midpoint of \( BC \). \( AE \) and \( BD \) are altitudes of \( \triangle ABC \).

3. Prove that two medians of an isosceles triangle have equal length.

4. In this triangle, the coordinates of \( D \) are \( \left( \frac{-8}{5}a, \frac{2}{5}c \right) \), and the coordinates of \( E \) are \( \left( \frac{8}{5}a, \frac{2}{5}c \right) \). Prove that two altitudes of an isosceles triangle have equal length.
Ordering Triangle Sides Lengths and Angle Measures

A. Write the angles in order from smallest to largest.
   If two sides of a triangle are not ____________, then the larger
   angle is opposite the ____________ side.
   Which side has the shortest length? _____
   What angle is opposite this side? _____ What is the smallest angle? _____
   Which side has the longest length? _____
   Which angle is opposite this side? _____ What is the largest angle? _____
   Write the angles in order from smallest to largest. ____________

B. Write the sides in order from shortest to longest.
   If two angles of a triangle are not ____________, then the
   longer side is opposite the ____________ angle.
   Find \( \angle K \). \[ \angle J + \angle K + \angle L = _____ \]
   Substitute known values. _____ + \( \angle K + _____ = 180^\circ \)
   Subtract to find \( \angle K \). \[ \angle K = _____ \]
   What is the smallest angle? _____ What side is opposite this angle? _____
   What is the shortest side? _____ What is the largest angle? _____
   What side is opposite this angle? _____ What is the longest side? _____
   Write the sides in order from shortest to longest. ____________

Applying the Triangle Inequality Theorem
Tell whether a triangle can have side lengths of 7, 12, 19.
Find the sum of the lengths of each pair of sides.

\[
\begin{align*}
7 + 12 &= _____ & 12 + 19 &= _____ & 7 + 19 &= _____ \\
\text{Is } 19 &> 19? _____ & \text{Is } 31 &> 7? _____ & \text{Is } 26 &> 12? _____
\end{align*}
\]

Is the sum of each pair of lengths greater than the third length? _____
Can these three lengths form a triangle? _____
The Triangle Inequality Theorem can be used to represent ranges of distances.

The distance from Hillary’s home to her dance studio is 4 miles. The distance from her home to her school is 7 miles. If the three locations form a triangle, what is the range of distances from her dance studio to her school?

Understand the Problem

1. What does “range of distances” mean?

2. The answer to this problem should be written as an __________.

Make a Plan

3. Sketch a diagram to represent the situation.

4. Let $x$ represent the distance between the dance studio and the ______________.

5. Apply the ______________________ Theorem to find the range of distances.

Solve

6. Set up three inequalities.

   $4 + 7 > _____$  
   $4 + x > _____$  
   $7 + x > _____$

7. Solve each inequality.

   $_____ > x$  
   $x > _____$  
   $x > _____$

8. Combine the inequalities. $_____ < x < _____$

9. The distance between Hillary’s school and her dance studio is greater than _____ miles and less than _____ miles.

Look Back

10. Choose a number between 3 and 11 to represent the distance between the school and the dance studio. Substitute the number into the Triangle Inequality Theorem and compare the lengths. For example, choose 5.

   Is $4 + 7 > 5$? _____  
   Is $4 + 5 > 7$? _____  
   Is $7 + 5 > 4$? _____
Using the Hinge Theorem and Its Converse

Use the diagrams to compare measures.

A. Compare \( \angle EDB \) and \( \angle ADB \).

The Converse of the Hinge Theorem states that if two
sides of one triangle are congruent to \___________ of
another triangle and the third sides are \________ congruent,
then the larger included angle is across from the \________ third side.

Compare the side lengths in \( \triangle EDB \) and \( \triangle ADB \).

\[ AD = \]  
\[ DB = \]  
\[ EB > \]  

Since \( EB > AB \), \( \angle EDB \) \________ \( \angle ADB \).

B. Compare \( AB \) and \( AC \).

According to the Hinge Theorem, if two sides of one
triangle are \________ to two sides of another
triangle and the \________ angles are not congruent,
then the longer third side is across from the \________ included angle.

Compare the sides and angles in \( \triangle ABD \) and \( \triangle ACD \).

\[ BD = \]  
\[ AD = \]  
\[ m\angle ADB \] \________ \( m\angle ADC \)

Since \( m\angle ADB \) \________ \( m\angle ADC \), \( AB \) \________ \( AC \).
Ready to Go On? Skills Intervention
5B
5-7 The Pythagorean Theorem

Find this vocabulary word in Lesson 5-7 and the Multilingual Glossary.

**Using the Pythagorean Theorem**
Find the value of \( x \). Give your answer in simplest radical form.

State the Pythagorean Theorem. \( \text{____} + b^2 = \text{____} \)

Which variable does the \( x \) represent in this problem, \( a \), \( b \), or \( c \)? \( \text{____} \)

\[ a^2 + b^2 = c^2 \]
\[ \text{____}^2 + \text{____}^2 = x^2 \]

Substitute 56 for \( a \), 36 for \( b \), and \( x \) for \( c \).

\[ \text{____} = x^2 \]

Simplify.

\[ \sqrt{\text{____}} = x \]

Find the positive square root.

\[ \sqrt{(16)\text{____}} = x \]

Simplify the radical.

**Identifying Pythagorean Triples**
Find the missing side length. Tell if the side lengths form a Pythagorean Triple.

A set of three nonzero whole numbers \( a, b, \) and \( c \), such that

\[ a^2 + b^2 = c^2 \]

is called a __________________________.

Which variable do you need to solve for in this problem, \( a \), \( b \), or \( c \)? \( \text{____} \)

\[ a^2 + b^2 = c^2 \]
\[ \text{____}^2 + b^2 = \text{____}^2 \]

Substitute known values.

\[ \text{____} + b^2 = 625 \]

Evaluate the powers.

\[ b^2 = \text{____} \]

Isolate the variable.

\[ b = \sqrt{\text{____}} \]

Take the square root of both sides.

\[ b = \text{____} \]

Solve for \( b \).

Do the side lengths form a Pythagorean Triple? Why or why not?

____________________________________

____________________________________
The Pythagorean Theorem gives you a way to find unknown side lengths when you know a triangle is a right triangle.

A moving company uses an 8-foot ramp that extends from the bottom of the truck to the ground. The distance from the bottom of the truck to the ground is 3 feet. How far along the ground does the ramp extend? Round to the nearest tenth of a foot.

Understand the Problem
1. What kind of triangle is formed by the ramp, the truck, and the ground? ______________
2. In a right triangle, you can use the ______________ ______________ to find the lengths of missing sides.
3. The ramp is the ______________ of the triangle. The distance from the bottom of the truck and the ground, and the distance along the ground are the __________ of the triangle.

Make a Plan
4. Sketch and label a triangle to represent the situation.
5. State the Pythagorean Theorem. ______________

Solve
6. Substitute the known values into the Pythagorean Theorem.
   \[ \Box^2 + x^2 = \Box^2 \]
7. Simplify and solve the equation. Round your answer to the nearest tenth of a foot.

8. How far along the ground does the ramp extend? ______________

Look Back
9. Substitute your answer into the Pythagorean Theorem and simplify.
   \[ 3^2 + \Box^2 = 8^2 \rightarrow \Box + \Box = 64 \rightarrow \Box = 64 \]
10. Is your answer reasonable? Explain.
**Finding Side Lengths in a 45°-45°-90° Triangle**

Find the value of $x$. Give your answer in simplest radical form.

Another name for 45°-45°-90° triangle is an ______________
______________ triangle.

By the Triangle Sum Theorem, the measure of the third angle
of this triangle is _____ . The triangle is a ______________ triangle.

$\overline{AB}$ is a _____ of this right triangle. _____ is the hypotenuse of this right
triangle.

In a 45°-45°-90° triangle, both legs are ____________ and the length of
the hypotenuse is the length of a leg times _____.

$BC = (_____ ) \overline{AB}$. Substitute 17 for $\overline{AB}$ to find $BC$.

$BC = _____$

**Finding Side Lengths in a 30°-60°-90° Triangle**

Find the values of $x$ and $y$. Give your answers in simplest radical form.

In a 30°-60°-90° triangle, the length of the longer leg is
the length of the shorter leg times ______. In this triangle,

$x$ represents the length of the ____________.

Longer leg = $\sqrt{3} ($shorter leg)

= (____) $\sqrt{3}$  Substitute known values.

$\frac{7\sqrt{3}}{x\sqrt{3}}$  Divide both side by $\sqrt{3}$.

_____ = $x$  Simplify.

In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the
______________ . In this triangle, $y$ represents the length of the ____________.

Hypotenuse = 2(shorter leg)

= 2(____)  Substitute known values.

$y = _____$  Simplify.
A musical triangle is an equilateral triangle with a side length of 4 inches. What is the height $h$ of the triangle? Round to the nearest tenth of an inch.

**Understand the Problem**

1. What is the measure of each angle of an equilateral triangle? ____
2. When you sketch the height of the triangle, what kind of triangles are formed?

**Make a Plan**

3. Complete the sketch and label the diagram that represents the situation.
4. The length of the side of the musical triangle represents the __________ of the 30°-60°-90° triangle.

**Solve**

5. Find the length of the shorter leg. ____
6. The length of the longer leg is _____ times the length of the shorter leg.
7. Find the length of the longer leg. Round your answer to the nearest tenth of an inch. ________________
8. What is the height of the triangle? ________________

**Look Back**

9. Substitute the known values into the Pythagorean Theorem and simplify.

\[
a^2 + b^2 = c^2
\]

\[
2^2 + \square = 4^2
\]

\[
4 + \square = 16
\]

\[
16.25 \approx 16 \quad \text{Is your answer reasonable? _____}
\]
1. Write an indirect proof that an obtuse triangle cannot have a right angle.

2. Write the angles of $\triangle WXY$ in order from smallest to greatest.

3. Write the sides of $\triangle MNO$ in order from shortest to longest.

Tells whether a triangle can have sides with the given lengths. Explain.

4. 11.7, 7.11, and 17.1

5. $a^2$, 4a, a, when $a = 5$

6. The distance from Tory’s home to the mall is 14 km. The distance from her home to the bus station is 8 km. If the three distances form a triangle, what is the range of distances from the school mall to the bus station?
5-6 Inequalities in Two Triangles

7. Compare \( AB \) and \( ST \).

8. Compare \( \angle XWF \) and \( \angle ZYW \).

9. Find the range of values for \( x \).

5-7 The Pythagorean Theorem

10. Find the value of \( x \). Give the answer in simplest radical form.

11. Tell if the measures 8, 9, and 15 can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

12. A park developer wants to put a bike trail from one corner of a rectangular park to the opposite corner. What will be the length of the trail? Round to the nearest yard.

5-8 Applying Special Right Triangles

13. A decorative platter is an equilateral triangle with side lengths of 14 inches. What is the height of the platter? Round to the nearest inch.

Find the values of the variables. Give your answers in simplest radical form.

14.

15.

16.
Relationships in Triangles
For Exercises 1–2, tell whether a triangle can have vertices with the given coordinates. Explain.

1. \( \triangle PQR \) has vertices \( P(3, 11), Q(1, 3) \) and \( R(5, 5) \).

2. \( \triangle JKL \) has vertices \( J(-5, 1), K(4, 2) \) and \( L(11, -1) \).

Answer each question.

3. A right triangle has legs with lengths \( x \) and \( 3(x + 1) \), and hypotenuse \( 4x - 3 \). Find \( x \) and the lengths of each side.

4. The figure at the right is drawn to scale. Compare \( BC \) and \( AD \). Which segment is longer? Explain your answer.

5. In the figure at the right, \( \measuredangle RPS = 53^\circ \). \( \overline{PQ} \parallel \overline{RS} \), and \( PQ = PR \). Compare \( QS \) and \( RS \). Explain your answer.

6. \( \triangle MNP \) is an equilateral triangle. \( \overline{RM} \equiv \overline{RP} \). \( MQ = 3\sqrt{2} \). Find the following:
   
   \( NQ \) ______
   
   \( MN \) ______
   
   \( RP \) ______